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Distorted wave response of ultrasonic annular stator incorporating non-uniform geometry

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HIGHLIGHTS

- The tooth geometry creates perturbation to the eigensolution of the periodic stator.
- The annular support affects the eigenvalue significantly for lower-order vibration.
- Tooth occupation and height affect the distortion more heavily than tooth count.
- Optimal tooth height for driving speed is obtained for different stator thickness.
- Traveling and standing waves coexist in ultrasonic motors like electrical motors.

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ABSTRACT

The traveling wave ultrasonic stator is normally fabricated with teeth. The tooth geometry improves the driving speed, but it creates natural frequency splitting and mode contamination, especially a distorted traveling wave. A dynamic model of a stepped-plate periodic stator is developed to examine the distortion. The stator is treated as an annular supported by a thin mid plate, and the support stiffness is formulated by using equivalent energy principle. The effects of the tooth and mid plate on the natural frequency and vibration mode are examined by using the perturbation method. The rules governing the frequency splitting, frequency perturbation as well as mode contamination are also identified. The traveling wave response and elliptical trace on stator surface are obtained by using the mode superposition method and they are proved to be distorted due to the tooth geometry. The response at the repeated doublets becomes coupled forward and backward traveling waves, but that at the split doublets becomes coupled forward traveling, standing and backward traveling waves. The results indicate that the tooth mass instead of the stiffness decreases the vibration amplitude and driving speed of the dominant wave, but their effects are different at the repeated and split doublets. Inspection of the model implies that the distortion can be suppressed by using a suitable combination of the wavenumber, tooth count, tooth height and occupying fraction. Numerical calculations are carried out to demonstrate the tooth geometry effect on the transient waveform, driving speed and elliptical trace. The optimization of the tooth geometry that can help achieve a purer traveling wave is discussed.

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1. Introduction

The traveling wave ultrasonic motor is a type of actuator powered by high-frequency oscillation of the electrically actuated piezoelectric element. As a key component, the stator absorbs energy from the piezoelectric element and creates traveling wave on its surface. The waveform is a primary concern because it directly drives the rotor pressed against the stator.

The previous studies normally assumed the stator to be a smooth vibrator [1–4]. With this assumption, a particular nodal diameter mode can be excited by the piezoelectric standing wave excitations with predetermined time and spatial phases, and a pure forward or backward traveling wave can be generated on the stator [2,5,6]. The above analysis is normally based on the superposition constructed in a model-free manner, and in particular the predicted response does not include any harmonics due to the smooth geometry assumption.

However, the tooth geometry is usually circumferentially designed on the stator, which makes the stator deviate from the axisymmetry. Because of this, the simple superposition can be invalidate because the deviation creates a rotationally periodic stator and causes problems like the natural frequency splitting and mode contamination [7–12]. Thus more accurate modeling and improved analysis are essential when predicting the dynamic behaviors. Wang et al. [7] and Kumar and Krousgrill [8] developed analytical models and examined the ultrasonic stator by means of the perturbation method based on the ring- and disk-shaped stators, respectively. For a similar structure, Wickert et al. [10] have approached the modal problem by using the direct perturbation method and achieved valuable results. They also predicted the responses of the coupled traveling and standing waves and verified them by experiment [11]. To accurately estimate the dynamic behaviors, Wang et al. [13] improved the superposition method and employed it to deal with the wave response of the rotationally periodic stator. The results were well compared with those from the perturbation and superposition methods [10]. It can be found that previous studies examined the contaminating waves at those orders different from the dominant one.

Apart from the circumferential geometry, the radial geometry can also be non-uniform, for example the stepped-plate stators in the USM 60 motor [2] and the Shinsei product [14]. Such stator mainly consists of three parts: the outer annular, the thin mid plate, and the inner fixed base. The previous modeling was usually based on the smooth plate [15,16] or ring [3,4,17,18], and for the purpose of simplification the annular was modeled as a straight beam rarely considering the support from the mid plate. Zhao [2] developed a semi-analytical electromechanical model by using the substructure method, where the circumferential and radial geometries were incorporated. Valuable insights into the dynamic response were gained and the effect of non-uniform geometry was proved to be significant. More simplified beam model including the circumferential and radial geometries is needed to well capture the actual and distorted wave response.

The ultrasonic stator is generally recognized as a case subjected to traveling load, just like those in Ref. [19]. This work targets the stepped-plate periodic stator stimulated by the piezoelectric traveling excitations. The tooth effect is treated as periodic mass and stiffness attached to a smooth stator, and the thin mid plate is modeled as an equivalent support. The perturbation and superposition methods [10–12] are employed to address the eigenproblem and traveling wave response. The effect of the tooth on the amplitude perturbation to the dominant mode is considered. The influences of the parameters including the tooth occupation, tooth count and tooth height, on the waveform and driving speed are examined.

2. Analytical modeling

Fig. 1 illustrates a stepped-plate ultrasonic stator consisting of an outer annular, mid plate and inner base, where the mid plate is connected with the annular and fixed base. The annular is modeled as a Bernoulli–Euler beam bending like a straight beam neglecting the curvature effect. Since the mid plate is thin and usually located at the neutral plane of the annular, it can be simplified as a support during the out-of-plane bending. An inertial frame $o - r\theta z$ is developed for the modeling, where the axis is directed toward the center of the first tooth.

2.1. Equivalent mass and support stiffness

The mid plate is assumed as an axial-oriented elastic support on the outer annular. A segment of angle $\mathrm{d}\theta$ is shown in Fig. 2, where the inner section A is fixed; the outer section B is free, and F is an axial force on section B. The inertia contribution of the mid plate is taken into account by means of an equivalent mass on per unit radian, $m_\mathrm{e} = \rho L h_\mathrm{c}(R_1 + R_2)/16$ (1/8 mass of a section with unit radian for approximation), where ρ denotes density, $L = R_2 - R_1$, and h_c is the height of the mid plate. The equivalent stiffness of the mid plate can be obtained by the relationship between the force and the deflection of section B on the trapezoidal cantilever.

With non-uniform width, the moment of the section inertia at coordinate x is

$$I_x = I_\mathrm{A}\left(1 + \frac{\eta}{L}x\right) \tag{1}$$

where I_A denotes the moment of inertia of the inner section A and $I_\mathrm{A} = R_1 h_\mathrm{c}^3 \mathrm{d}\theta/12$ and $\eta = R_2/R_1$.