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On attenuation of free and forced waves in an infinitely long visco-elastic layer of a constant thickness

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HIGHLIGHTS

- A hierarchy of reduced order models is employed to analyse wave propagation in a viscoelastic layer.
- The measures of attenuation of free and forced waves are proposed and compared with each other.
- A high sensitivity of the attenuation levels to excitation conditions at high frequencies is demonstrated and explained.

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ABSTRACT

The conventional concepts of a loss factor and a complex-valued elastic module are used to study wave attenuation in a visco-elastic layer. The hierarchy of reduced order models is employed to assess attenuation levels of free and forced waves in various situations. First, the free waves are considered. In the low frequency limit, the attenuation of these waves is found to be in the excellent agreement with the existing knowledge. At high frequencies, predictions of the reduced order models fully agree with the solutions of exact Rayleigh–Lamb problem. Alternative excitation cases are considered for the forcing problem and a measure of the attenuation level is proposed and validated. The differences between two regimes, the low frequency one, when a waveguide supports only one propagating wave, and the high frequency one, when several waves are supported, are demonstrated and explained.

1. Introduction

The assessment of origins and actual levels of damping has always been a challenging task in various applications such as structural intensity analysis and non-destructive structural health monitoring. As a manifestation of uncertainties in this assessment, commercially available finite element software typically leaves to a customer the choice of parameters defining the extent to which the damping matrix is proportional to the stiffness and to the mass ones.

On the other hand, the quantification of viscous damping in linear one-degree-of-freedom systems is truly elementary and based on simple energy considerations, which bring to light a concept of the loss factor. Therefore, it has long been attempted to extend this quantification to multi-degree-of-freedom and continuous systems. This paper goes along these lines and is concerned with assessment of the attenuation of time-harmonic waves in an infinitely long visco-elastic layer of a constant thickness. The goal is to demonstrate to what extent the loss factor, generally recognised as a classical measure of the energy dissipation in mechanical systems of finite dimensions, is applicable to estimate spatial and temporal attenuation of waves in visco-elastic structures.

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For the longitudinal (i.e., the first symmetric) and the flexural (i.e., the first skew-symmetric) waves, this has been done in the classical text [1] by means of the elementary Bernoulli–Euler theory known to be asymptotically consistent in the low frequency — long wave limit with the exact Rayleigh–Lamb solution, see [2]. In fact, the wave attenuation analysis reported in [1] serves as a theoretical background to link the material loss factor and the imaginary part of Young's module. However, such an assessment has not yet been done for the two-mode (i.e., Timoshenko and Herrmann–Mindlin) models as well as for higher-order models. This paper is aimed to fill in this gap and to give a systematic account of the free waves' attenuation in the framework of simplified theories verified against the exact solution. Alongside, an assessment of wave attenuation in the case of a forced response of an infinite waveguide, which supports several propagating waves, constitutes the research goal of this paper. To the best of the authors' knowledge, these issues have not yet been fully addressed in the literature.

The paper is structured as follows. Section 2 is concerned with discussion of the concepts of the loss factor and complex-valued Young's module. In Section 3, the brief exposition of the canonical Rayleigh–Lamb problem is presented and a hierarchy of reduced order models of wave propagation in a layer is described. In Section 4, the alternative measures of attenuation of free waves are discussed. This section is also concerned with comparison of such attenuation predicted by low order theories and in the exact formulation. In Section 5, an attenuation measure for the forced response is proposed and used in the framework of the theories presented in Section 3. Attenuations of free and forced waves are compared with each other in the representative frequency ranges and the influence of excitation conditions is explored. The findings are summarised in conclusions.

2. The loss factor and complex-valued Young's module

As the wave attenuation is caused by the internal material damping in a structure exhibiting steady state cyclic deformation, it is natural to characterise the energy dissipation in terms of energy quantities. The loss factor, defined as a ratio of the energy dissipated per oscillation cycle N_D and the total energy N, see [1,3,4]: $\eta = \frac{N_D}{2\pi N}$, is such a measure, originally introduced for single-degree-of-freedom systems. Since any linear oscillatory system may be decomposed into a set of uncoupled oscillators, loss factors are commonly used in the modal format, i.e., to assess the level of attenuation at each mode of vibration of a multi-degree-of-freedom or continuous system of finite dimensions. The point to be addressed in this paper is how the concept of a modal loss factor is adjusted in a waveguide theory for calculation of the level of attenuation of otherwise (i.e., in the absence of viscosity) propagating free waves, not necessarily in the low frequency — small wavenumber limit.

It should be noted that Refs. [5–7] are concerned with the estimation of loss factors for waves propagating in laminated structures by means of the wave finite element method. The comprehensive literature survey on the damping estimation in the framework of the finite element method is also presented in [6]. However, the modal strain energy method as formulated in Eq. (10), p. 3931 in this reference gives the assessment of attenuation of an individual free wave. In what follows in the present paper, a model, much simpler than those introduced in abovementioned references, is considered to facilitate a rigorous analysis of attenuation of free and forced waves based on the canonical definition of the loss factor. The authors are of the opinion that such an analysis will serve as a novel useful supplement to the findings reported in [5–7].

Steady state harmonic vibrations hereafter are treated with the time-dependence of all state variables chosen in the form $\exp(-\mathrm{i}\omega t)$. This modelling brings to light the complex-valued Young's module [1] with (given the stated above sign convention) the negative imaginary part, $\hat{E} = E - iE_i$. It is in common practice to present the imaginary part of the complex-valued Young's module via a loss factor in the form $E_i = \eta E$. Here the loss factor η is considered as frequency-independent (see [1, p. 153]). This model is widely used in the conventional harmonic finite element analysis, where the complex-valued stiffness matrices have been introduced and used in many research papers with [6–8] being just a few examples. Furthermore, the commercially available finite element software, such as ANSYS, utilises this concept [9]. Classical Refs. [1,3,10–12] explain pros and cons of the model used hereafter and provide the detailed discussion of damping mechanisms. In the classical theory of linear wave propagation in viscoelastic materials, introduction of the complex-valued Young's module entails introduction of complex-valued propagation speeds: 'If one substitutes the modulus of elasticity $\bar{D} = D'(1 + \mathrm{i}\eta)$ into the wave equation, one finds that the propagation speed becomes complex' ([1, p. 155], the third and fourth lines from the bottom, time dependence is taken as $\exp(\mathrm{i}\omega t)$). The concept of a complex-valued propagation speed has been adopted in several publications [13–17] and implemented in SAFE (semi-analytical finite element) method. We employ this concept in an analytical solution of the canonical Rayleigh–Lamb problem.

3. The low-order models of wave propagation in an elastic straight layer of constant thickness

We consider an infinite elastic layer in the plane strain state with the traction-free boundary conditions, as shown in Fig. 1(a). Analysis of the wave propagation in such a layer constitutes the classical Rayleigh–Lamb problem. Due to the symmetry in the boundary conditions at $y_{\mathrm{dim}} = \pm\frac{h}{2}$, the symmetric and skew-symmetric modes (see Fig. 1(b)) are decoupled from each other. The conventional spatial dependence is taken as $\exp(\mathrm{i}k_{\mathrm{dim}}x_{\mathrm{dim}})$, so that the dispersion equation for free symmetric modes is [2, p. 223]:

$$\frac{\tan\left(\frac{q}{2}\right)}{\tan\left(\frac{p}{2}\right)} = -\frac{4k^2pq}{\left(q^2 - k^2\right)^2}. \tag{1}$$