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Electrohydrodynamic pulsatile flow in a channel bounded by porous layer of smart material

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ABSTRACT

Electrohydrodynamic dispersion due to pulsatile flow in a channel bounded by porous layer of smart material is studied considering both steady and unsteady cases using both BJ and BJR-slip conditions. We found that in the case of steady flow, the dispersion coefficient, D_s^* decreases with an increase in electric number W_e but increases with an increase in porous parameter σ_p in the case of BJ-slip condition. However this nature is different in the case of BJR-slip condition in the sense that the dispersion coefficient, D_s^* increases for certain values of W_e and then decreases with an increase in W_e. In the case of unsteady flow, the dispersion coefficient, D_u^*, decreases with an increase in W_e and σ_p for both BJ and BJR conditions. In particular, we found that the value of D_s^* for steady flow in the case of BJ-slip condition is less than that of unsteady flow. The opposite is true for BJR condition. The findings are useful in the design of robust and efficient artificial organs in the human body.

1. Introduction

Solutions for pulsatile fully developed hydrodynamic (Boussinesq [1], Berker [2], Drake [3], Tsangaris and Vlachakis [4]) and magnetohydrodynamic (Narsimhan [5], Rudraiah [6]) flows in a rectangular channel or a tube have been investigated because of their importance in science, engineering and technology particularly in blood flows. The problems investigated in the literature mentioned above are concerned with pulsatile flow in a channel bounded by impermeable boundaries. In medical sciences, for example dialysis of blood in artificial kidneys, pumping of blood in arteries bounded by endothelium, the fluid in the synovial joint bounded by cartilages involve the channels bounded by porous layers and hence the results obtained in the literature mentioned above are not of much use to such problems in medical sciences. It is known that cholesterol and other fat substances will be dispersed to the body through the endothelium in arteries and hyaluronic acid (HA), glycoproteins and other nutrients will be dispersed from synovial fluid to cartilages. The literature on pulsatile flow cited above is also independent of the study of dispersion of cholesterol, fat substances and other nutrients using the dynamics of pulsatile flow. We note (Rudraiah [7], Ng et al. [8]) that in these problems of medicine, the presence of glycoproteins, fixed charge density and so on in the blood is regarded as poorly conducting fluid whose nature will depend on the approximations connected with channel width and corpuscular scale (Zamir [9]). If the channel width is larger than the corpuscular scale then the purely conducting fluid behaves as a Newtonian fluid having linear constitutive equations for the Chauchy stress. The study of such Newtonian poorly conducting fluids in the presence of an electric field is called electrohydrodynamics (EHD). However, if we relax this assumption of channel width, that is if the channel width is of the same order of corpuscular scale, then such poorly conducting fluids will have nonlinear constitutive equations. In such situations the

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poorly conducting fluids in the presence of an electric field are called electrorheological fluids which may change rapidly their properties; in particular their viscosity can increase significantly. Such fluids can behave like semi-solids or even like solids in the presence of a strong strength of electric field. Rajagopal [10] has proposed a constitutive law suitable to such electrorheological fluids. The constitutive equations and their implications in three dimensional models have been introduced by Rajagopal and Wineman [11] by assuming the electric field is uniform. Later, Rajagopal and Ruzicka [12] have relaxed this assumption of uniform electric field and studied this problem using variable electric field determined using the Maxwell equations. Recently Rajagopal and Ruzicka [13] have made a detailed study of the thermodynamics of electrorheological fluids.

The electrorheological fluids becoming solids in the presence of a strong electric field (see Busuioc and Cioranescu [14]) have many industrial applications like braking devices, control of vibrations and soon. Under such situations the rheological fluids may not flow if the electric field is increased. However, in biomechanical problems discussed above there will be flow of poorly conducting fluid that is EHD flow, to disperse the nutrients, proteins, fat substances and so on through porous nature of cartilages in synovial joints, the endothelium in arteries and so on. Therefore, the aim of this paper is to study the dispersion phenomena due to pulsatile flow using Newtanian nature of EHD and we study the dispersion phenomena using electrorheological fluids in the future.

In a poorly conducting flow, the electrical conductivity is a strong function of concentration where the difference in conductivity due to concentration releases free charges in the fluid which in turn produces the electric field known as concentration electric field (Rudraiah [7], Rudraiah and Ng [8]). This electric field produces not only the current density which acts as sensor and also produces a force which acts as actuator which are the two basic properties of smart materials. One of the advantages of smart systems is to trace their origin to the field of research that could mimic human blood, muscular and nervous system. The other main advantages of smart materials, synthesized by considering poorly conducting fluid, in these applications of medical science are to produce the robust and effective artificial organs like artificial kidneys, cartilages in synovial joints (Ng et al. [8,15]) and human heart using non biological system, which will achieve the optimum functionality observed in medical systems through emulation of their adaptive capabilities and integrated designs. To mimic the characteristics of natural organs explained above, it is essential to understand the dynamics of pulsatile flow and its effect on the dispersion of cholesterol and other fat substances from blood in arteries through porous nature of endothelium bounding the arteries and the dispersion of hyaluronic acid (HA), glycoproteins and other nutrients from synovial fluid to porous nature of cartilages in synovial joints. In the literature, different dispersion phenomena like Taylor dispersion valid for large time, generalized dispersion valid for all time (Rudraiah and Ng [8]) are studied. The effective functioning of the artificial organs, in the problems of medical sciences mentioned above, depends on the dispersion phenomena in two regions, one in a poorly conducting fluid saturated porous layer which mimics either endothelium in arteries or cartilages in synovial joints and the other is a poorly conducting fluid flow in a channel or tube called free flow with an interface, called nominal surface (Beavers and Joseph BJ [16], Rudraiah [17]), between the porous layer and free flow. At this nominal surface, there will be a slip velocity due to momentum transport from free flow to the porous layer where the porous layer is saturated resulting in converting momentum into a drag known as BJ slip. This BJ ([16], see (3.7) slip condition is independent of the thickness of the porous layer and hence valid when the thickness of the porous layer is much larger than the thickness of free flow, as in many geophysical applications. To overcome this restriction, Rudraiah [17] has proposed a new slip condition which is now known as BJR-slip condition. We note that since the BJR [17] slip condition (see Eqs. (3.22) and (3.23) below) involves the thickness of the porous layer, H, and Hence as $H \to \infty$, it reduces to BJ-slip condition. This BJR-slip is valid when the thickness of the porous layer is smaller or comparable with the thickness of the free flow which is relevant to the biomechanical problems mentioned above.

Therefore to mimic the dynamics of pumping of fluid and the corresponding dispersion phenomena in biological organs mentioned above, we have to consider pulsatile fully developed poorly conducting flow in a channel bounded on both sides by densely packed porous layer in the presence of an electric field. The dispersion phenomena in electrohydrodynamic pulsatile fully developed poorly conducting flow either in a rectangular channel or in a tube has not been given any attention to our knowledge inspite of its importance in the manufacture of artificial organs in medical applications mentioned above. The study of it is the main objective of this paper by considering Taylor [18] model. We note that the result obtained in this paper will also be of importance in the study of dispersion of natural pollution in air and water, combustion processes, chemical industries, fluidisation, flow in rockets and so on.

To achieve the objective of this paper, the plan of the paper is as follows. In Section 2, we discuss the basic equations, the corresponding boundary conditions in a channel bounded by a densely packed porous layer. The dispersion of proteins and other substances by steady pulsatile flow is studied in Section 3 following Taylor model with BJ and BJR slip boundary conditions. In Section 4, the dispersion due to unsteady pulsatile fluid flow is discussed using Taylor model with BJ and BJR slip conditions. The important conclusions are drawn in the final section.

2. Mathematical formulation

To investigate the effect of pulsatile flow on electrohydrodynamic (EHD) dispersion in a poorly conducting fluid in the presence of an electric field we consider a physical configuration as shown in Fig. 1.

It consists of a rectangular channel of width $2h$ through which a poorly conducting incompressible fluid flows between two densely packed poorly conducting fluid saturated porous layers each of width H at $y = \pm h$. The poorly conducting fluid