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A fractal study of sound propagation characteristics in roughened porous materials

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HIGHLIGHTS

- Sound propagation characteristics in roughened porous materials is investigated.
- Fractal geometry theory is employed to simulate rough surface topography.
- Sound propagation model in roughened porous materials is built.
- Sound absorption coefficient is improved because of existing of interface roughness.

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ABSTRACT

The influence of rough elements of porous materials on sound propagation characteristics in roughened porous materials is investigated. Rough surface topography on the roughened porous materials is simulated by the fractal geometry theory, in which relative roughness is defined clearly as a function of the fractal dimension, porosity, average diameter and diameter ratio of the hexagonal elements on pore wall of the porous materials. Based on the sound propagation model of the smooth porous materials, a sound propagation model of the roughened porous materials is built. The effective density and bulk modulus, acoustic impedance and propagation constant, flow resistivity and sound absorption coefficient of the roughened porous materials are derived and discussed as a function of the relative roughness. It is demonstrated that the sound absorption coefficient of the roughened porous materials is improved as the relative roughness increases. The model predictions for the sound absorption coefficient of the roughened porous materials are well agreed with that of the existing test results.

1. Introduction

Porous materials, which possess a lot of unique multifunctional attributes with high specific strength/stiffness, high heat dissipation and low weight, are widely used in a diversity of sound absorption applications. However, the porous materials usually show poor sound absorption properties [1,2]. In order to enhance the sound absorption properties of the porous materials, several methods have been attempted, including compressing the porous materials to increase tortuosity of cells [3], perforating on the porous materials to form double porosity materials [4] and particularly growing micro-rods on cell walls of the porous materials [5,6]. It is generally demonstrated that these methods are used to improve surface roughness on the cell wall in different ways. In the present study, the surface roughness of the porous materials through

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growing micro-rods is analyzed by the fractal geometry theory and the sound propagation properties of the roughened porous materials are investigated.

Due to the complication of the structure of the porous materials, a semi-phenomenology theoretical model of the porous materials is proposed by Zwikker and Kosten [7] and Biot [8]. More recently, Allard [9], Stinson and Champoux [10] have modified the theoretical model to make it more applicable. It is known that acoustic wave dissipation through the porous materials is caused by the viscous friction and thermal conduction on the pore wall between the matrix and air. The sound absorption characteristics of the porous materials are related to propagation behavior of air inside the cell, and are strongly dependent on the surface morphology of the cell structure which is determined by fabrication methods [3–6].

The disordered nature of the cells in the porous materials and the clusters in micro-pores suggest that the cell distribution and numerous clusters satisfy the fractal nature [11–14]. For the microstructures, these cells and their distributions are analogous to cells in sandstone [15], to islands or lakes on earth [16], and to contact spots on engineering surface [17,18]. Some investigators [19–21] show that rough elements on cell surface are statistically isotropic and self-similar fractal. Therefore, we also assume that the rough elements on a rough surface do not overlap each other and are statistically self-similar fractal. That is, they look the same on all scales.

The fractal geometry is originally founded by Mandelbrot [16] to describe disordered objects using fractal dimensions. Chen et al. [22] describe the topography of the rough surface by using a Cantor set to numerically simulate heat transfer in a micro-channel. Warren and Krajcinovic [17] apply the random Cantor set to simulate the elastic-perfectly plastic contact of the rough surface. Chen and Cheng [23] measure the fractal dimension for rough surface profiles in a micro-channel, and find that the Poiseuille number in the roughened micro-channel is a function of the classical Poiseuille number, average height of rough elements and hydraulic radius. Chen et al. [24] apply the W-M function to characterize the multiscale self-affine roughness. The lattice Boltzmann method is applied to investigate the gas flow in a micro-channel with rough surface characterized by the fractal geometry [25]. Assuming the rough surface in a micro-tube as a Gaussian distribution, Bahrami et al. [26] investigate the influence of wall roughness on the fully developed laminar flow in a rough micro-tube.

The surface roughness is described by the geometric product specification standards, average surface roughness, mean value of single roughness depth [27] and root-mean-square deviation [28]. Li et al. [29] define the relative roughness as the average height of the roughness profile. Pfund et al. [30] define the relative roughness in semi-roughness channel, involving in the average and maximal height of roughness profile, respectively. Recently, Yang et al. [11] propose a theoretical model to define the relative roughness, in which the every parameter has clear physical meaning and reveals more mechanisms affecting the average height on the rough surface.

In this work, in view of the assumption of hexagonal prism rough element, the effective average height of the rough elements is derived by the fractal geometry. Based on the sound propagation model of the smooth porous materials, a sound propagation model in the roughened porous materials is also derived and the flow resistivity ratio is obtained. The sound propagation characteristics of the roughened porous materials are analyzed and compared with that of the smooth porous materials.

2. Fractal theory for roughened porous materials

The cumulative size distribution of islands on earth's surface follows fractal scaling law [16]: $N(A > a) \approx a^{-D/2}$, where N is the total number of islands of area A greater than the spot area a, and D is the fractal dimension of the surface. Majumdar and Bhushan [18] use the fractal scaling law to describe the contact spots (roughness elements) on engineering surface, and the fractal scaling law is rewritten as $N(L \geq a) = (a_{\max}/a)^{D/2}$, where a and $a_{\max}$ are the spot area and the maximum spot area on the engineering surfaces, respectively, L is the scale of measurement and $0 < D < 2$.

As shown in Fig. 1(a) and (b) [5], the rough elements on surfaces of the porous materials are assumed as regular hexagonal prism. In view of Yang et al. [11], the ratio of the height h_i to the base diameter λ_i of the regular hexagonal prism is $\xi = h_i/\lambda_i$. Thus, the base diameter distribution of the hexagonal prism also follows the above fractal scaling law and is given [21,31] by

$$N(L \geq \lambda) = (\lambda_{\max}/\lambda)^{D} \tag{1}$$

where λ and $\lambda_{\max}$ are base diameter and the maximum base diameter, respectively, and D is the fractal dimension for the base diameter distribution of the hexagonal prism. Here, $D = 2$ means that a surface is so rough (in other words, a surface is covered so many rough elements) that the surface profile makes the diameter of the pore become as small as possible, i.e. the actual cross-sectional diameter for air flow in the porous materials becomes as small as possible, while $D = 0$ corresponds to a smooth surface because the number of rough elements on the surface is approximately zero.

It can be seen from Fig. 1(a) and (b) [5] that there are numerous rough elements on the roughened surfaces. So, Eq. (1) can be considered as continuous and differentiable equation. Differentiating with respect to λ, the number of rough elements in the infinitesimal range of λ to $\lambda + d\lambda$ can be found as [11]

$$-dN = D\lambda_{\max}^{D}\lambda^{-(D+1)}d\lambda. \tag{2}$$

Here, $-dN > 0$ and Eq. (2) indicates that the number of rough elements increases with the decrease of the base diameter.