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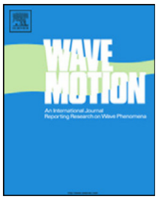

Modal analysis of the scattering coefficients of an open cavity in a waveguide

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HIGHLIGHTS

- The roles that frequency-dependent eigenmodes play in the acoustic scattering of an open cavity connected with rigid-wall ducts are revealed.
- Peaks and valleys of scattering coefficients are generally found to be result of constructive and destructive interferences between eigenmodes.
- An approximated equation is proposed to characterize the Fano resonance which is induced by quasi-trapped mode.
- The usage of frequency-dependent and frequency-independent eigenmodes for describing free and forced sound field is clarified.

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ABSTRACT

The characteristics of an acoustic scatterer are often described by scattering coefficients. The understanding of the mechanisms involved in the frequency dependent features of the coefficients has been a challenge task, owing to the complicated coupling between the waves in open space and the modes inside the finite scatterer. In this paper, a frequency-dependent modal description of the scattering coefficient is utilized to study the modal properties of the scatterer. The important role that eigenmodes play in defining the features of the scattering coefficients is revealed via an expansion of the coefficients by the eigenmodes. The results show the local extrema of the scattering coefficients can be attributed to the constructive/destructive interference of resonant and non-resonant modes. In particular, an approximated equation, which is equivalent to the standard Fano formula, is obtained to describe the sharp anti-symmetric Fano characteristics of the scattering coefficients. The special cases where scattering is dominated by a single resonance eigenmode, corresponding to the "resonance transmission", are also illustrated.

1. Introduction

The scattering coefficients of acoustic scatterers are important parameters as they are used to relate incident and scattered waves. Traditionally, sound scattering in a duct by a muffler is described by one-dimensional (1D) transmission line theory, and explained by the mismatch of the specific impedance at the inlet and outlet of the muffler [1]. To accommodate cross modes in a duct with scatterers, the wave matching technique [2,3] is used. The finite element and boundary element methods, are used to determine the scattering coefficients when the geometries of the ducts and scatterers are complicated [4,5]. While these wave matching and numerical methods are capable of producing accurate coefficients, they are less useful in directly delivering the physical insight into the peaks and valleys in the coefficient curves.

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An alternative approach, which is motivated by the observation of trapped and quasi-normal modes inside the scatterers, is to describe the scattering in terms of the coupling between the waves in the ducts and modes inside the scatterer. This was inspired by the early work of Flax et al. [6], where the sound scattering from submerged elastic bodies is affected by various kinds of interference between the resonance scattering at the eigenfrequencies of the vibration of the body and the rigid-body scattering. Recently, such approach has been used to explain the peaks and valleys in the transmission loss curve of an expansion chamber subject to an incident plane-wave in a 1D duct [7]. It was demonstrated that the characteristics (complex eigenvalues and mode shape functions) of the frequency-dependent quasi-normal modes of the expansion chamber [8] allow the correct expansion of the sound pressure in the chamber. The coupling of the quasi-normal modes with the incident and transmitted sound waves sheds some light on the transmission loss. For example, the minimum values in the transmission loss occur when the frequency of the incident wave equals the real part of the eigenfrequency of the quasi-normal mode. On the other hand, the maximum transmission loss appears at those frequencies where the superimposed contribution from the participated modal factors is at minimum. However, the extension of these previous works to two-dimensional (2D) or three-dimensional (3D) expansion chambers and modelling of the coupling between the quasi-modes in the chambers and the incident and transmitted waves including cross modes, is not straight forward. Furthermore, because of the involvement of cross mode components in the 2D and 3D configurations, extra coupling mechanisms due to participation of the cross modes may lead to a more complete understanding of the characteristics of the scattering coefficients.

Most recently, Maksimov et al. [9] and Lyapina et al. [10] looked at the aforementioned scattering problem from a more fundamental view of the acoustical properties of an open cavity. An open cavity is a finite acoustical space with defined boundary conditions at its internal wall (*e.g., a* rigid-wall condition) and some boundary areas open to infinite space(s), such as semi-infinite waveguides. Because of the energy exchange between the sound fields inside the open-cavity and the infinite space(s), the sound field inside the cavity is characterized by the non-Hermitian Hamiltonian. They used the acoustic coupled mode theory for the calculation of the frequency-independent eigensolutions of the open cavity and revealed a mechanism resulting in acoustic trapped modes in the cavity, namely Friedrich–Wintgen two-mode full destructive interference. Although being used to explain the occurrence of Fano resonance, the nonorthogonal properties and incompleteness of the frequency-independent eigensolutions of the open cavity make the direct modal interpretation of the transmission coefficient (which is a forced scattering problem at the frequency of an incident wave) a difficult task. Xiong et al. [11], on the other hand, derived the scattering matrix of the open cavities due to incident waves from one of the waveguide connected to the cavity. Frequency-dependent eigensolutions of the effective Hamiltonian matrix (including the interaction between cavity and connected waveguides) for the sound field in the cavity were used to describe the scattering coefficients and to explain the links between the eigenmodes and a trapped mode and the corresponding transmission zero. Their contribution makes possible an analysis of the scattering coefficients of the open cavities by using the frequency-dependent eigensolutions of the cavity.

In this paper, the method for frequency-dependent eigenmodes and the scattering matrix developed by Xiong et al. [11] is adopted. Instead of focusing on the Fano resonance induced by trapped modes, it is used to calculate and explain the general scattering features of the open cavities connected with waveguides (*e.g.,* conventional muffler configuration) and the roles played by eigenmodes in determining the frequency-dependent features of the scattering coefficients. Through numerical studies, it is revealed that extrema in scattering coefficients are *generally* a result of interference between eigenmodes, rather than the contribution from single resonant mode, as is traditionally assumed. The Fano resonance induced by highly localized modes (quasi-trapped modes), as observed by Hein et al. [12], is also revisited in terms of frequency-dependent eigenmodes. Finally, some remarks are made to clarify the usage of frequency-dependent and frequency-independent modes in conducting modal analysis in scattering problems.

2. Modal description of the scattering coefficients

This paper considers the scattering problem in a 2D acoustic scatterer (see Fig. 1), comprising a cavity connected by N uniform ducts. Omitting the time-dependence term, $e^{-i\omega t}$, the sound pressure field is governed by Helmholtz equation.

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)p(x,y)=0, \tag{1}$$

and corresponding boundary conditions, where k is the wavenumber. Since $k=\omega/c_0$ (where c_0 is the speed of sound), k will be used hereafter to represent source frequency, *i.e.*, whenever "frequency" is mentioned, it refers to the wavenumber. For the sake of simplicity, the rigid-wall boundary condition is assumed for the cavity and ducts. Although details of the coupled mode theory [9–11] have already been published, this paper provides a brief description of the derivation using coupled mode theory for the scattering coefficients of the scatterer in terms of frequency-dependent eigenmodes, for the convenience of the reader and to make the paper self-contained.

The first step is to express the sound pressure in terms of a local basis in different regions. The geometry is partitioned into $(N+1)$ regions: a closed cavity Ω_c and N semi-infinite ducts Ω_n $(n=1,2,\ldots,N)$. The pressure field in the nth duct is expanded into duct modes when taking a local coordinate (x_n,y_n) with x- and y-axes that are, respectively, perpendicular