Wave Motion 67 (2016) 102–115

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Direct numerical modeling of time-reversal acoustic subwavelength focusing

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HIGHLIGHTS

- We propose to reproduce Helmholtz resonance, band gaps and subwavelength focusing by a direct solution of the wave equation with the Spectral element method and a fine scale description of the media.
- The numerical results reveal that only very simple physics (the standard acoustic wave equation) and small scale scattering heterogeneities with a specific shape is required for the subwavelength focusing.
- The non-periodic homogenization upscaling tool can be useful to identify subwavelength focusing from a regular focusing at the diffraction limit but in a locally slower effective medium.

ARTICLE INFO

Article history:
Received 3 May 2016
Received in revised form 19 July 2016
Accepted 25 July 2016
Available online 30 July 2016

Keywords:
Subwavelength focusing
Numerical modeling
Time-reversal

ABSTRACT

Focusing waves back to their original source position is possible both experimentally and numerically thanks to time reversal mirrors (TRM). For a TRM placed in the far field of the source, the focusing spot of the reversed wavefield is subject to the diffraction limit and cannot be smaller than half the minimum wavelength, even for a very small source. Yet, numerous time reversal experiments in resonating media have shown subwavelength focusing. In this work, we show that it is possible to model these subwavelength focusing observations with simple physics, only the 2-D standard acoustic wave equation, and with specific fine scale heterogeneity. Our work is based on the spectral element method to solve the wave equation and to model time reversal experiments. Such a method makes it possible to propagate very long time series in complex and strongly discontinuous media with high accuracy. The acoustic wave equations are solved at the fine scale in media with one or more split rings of size much smaller than the wavelength. Such split rings produce a Helmholtz resonance effect as well as propagation band-gaps. We show that, in such media, even with a single split ring resonator, subwavelength focusing down to 1/13th of the minimum wavelength can be observed.

1. Introduction

Waves created by a localized source diverge from their origin and time reversal methods have the ability to focus these waves back to their original source location using a time reversal mirror (TRM) regardless how complex the media are,

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http://dx.doi.org/10.1016/j.wavemoti.2016.07.010
0165-2125/

provided that there is no attenuation. It is based on the theory of time reversal invariance and spatial reciprocity of the wave equations [1]. It has been widely studied in the acoustic [2–5], ultrasound acoustic [6–8], electromagnetic [9,10] and elastic wave domains [11,12]. In seismology, it has been used in seismic interferometry, earthquake source localization and reconstruction [13,14], or in tomographic imaging [15].

The diffraction limit is a well known wave phenomenon which can be observed in many circumstances such as refocusing of light by optical lens [16,17]. It states that any focusing and imaging resolution of acoustic, elastic or electromagnetic waves, has a resolution limit of $\lambda/2$, where λ is the wavelength. Indeed, wavefield spatial variations smaller than $\lambda/2$, such as evanescent waves in the near-field of a point source (see [1] page 72 for a precise definition of the source near-field), decrease very quickly from its origin and become negligible just a few wavelengths away. It can be shown that, for a TRM placed in the far field of a source (typically a few wavelengths away), regardless how spatially small the source is, the focusing spot at the source original location can never be smaller than roughly half of the wavefield wavelength [18]: this is the so-called diffraction limit. Nevertheless, one of the most astonishing observation about time reversal is the subwavelength focusing which, in some specific media, seems to beat the diffraction limit [19]. Recent works, such as "superlens" designed by metamaterial with negative index of refraction to enhance the evanescent waves in the near field [20–23], or "hypenlens" made of anisotropic metamaterial with hyperbolic dispersion relations to transfer the evanescent waves into propagating waves [24,25], show subwavelength imaging capacity.

In homogeneous media, the spatial resolution of the time reversal focusing spot is limited not only by the diffraction limit, but also the aperture of the TRM. However, when the medium between the source and the TRM is heterogeneous and strongly scattering, the focusing resolution will be enhanced significantly [26,27], but not beyond the diffraction limit. Nevertheless, Mathias Fink and his group achieved the subwavelength focal spot by using an "acoustic sink" in the time reversal refocusing process, although an active time-reversed source must be used to recover the evanescent waves [28]. A more meaningful experiment designed by [29] showed focal spots far beyond the diffraction limit ($\lambda/13$) in the electromagnetic domain with TRM placed in the far field, and a random distribution of scatterers placed in the near field of the focusing point. Following this work, it has been shown that strongly coupled subwavelength Helmholtz resonators, for example, an array of soda cans, can be very efficient to focus waves to the subwavelength scale from the far field [30,31]. The Helmholtz resonator has been reported to have negative effective bulk modulus, and can be used for the design of negative acoustic index metamaterials [32,33]. These experiments promise a wide range of applications in various fields such as telecommunications [34], underwater communications [35,36], optical imaging [37], sensing at higher frequencies [38,39].

The numerous observations of time reversal subwavelength focusing mentioned above are intriguing, especially for the soda cans experiment. Recent results show the same sharp focusing spot but with soda cans arranged in a different way and without using time-reversal [40]. However, the authors of this later recent work conclude that no sub-diffraction-limited focusing is observed if the diffraction limit is defined with respect to the wavelength of the guided mode in the metamaterial medium rather than the wavelength of the bulk wave in air. A complete understanding of the processes involved is not trivial and is bounded by observation and experimental limitations such as the one linked to the dissipation loss of the media, which reduce the resonance quality factor Q and finally influence the focus resolution and the position accuracy of the focus spot [41], or linked to the experimental difficulties to tune media parameters and to obtain fine observations.

The objective of this work is to show that it is possible to model and reproduce these observations with only the standard acoustic wave equation in 2-D but associated with a specific fine scale description of a heterogeneous medium producing Helmholtz resonance. To do so, we rely on the Spectral Element Method (SEM) to solve the acoustic equation in the time domain [42–45]. The spectral element method is a powerful tool which has the flexibility and accuracy to handle strongly discontinuous with complex unstructured meshes, meanwhile offering a spectral convergence with the polynomial degree, leading to a very low dispersion, and still very efficient thanks to its diagonal mass matrix, its explicit time scheme and parallel implementation. Numerical modeling makes it possible to flexibility control all the relevant parameters of the experiment and measure all physical quantities such as pressure or particle displacement anywhere in the domain, opening the door to a better understanding of this phenomenon.

The article is organized as follows: in Section 2 the basic theory of time-reversal acoustics and its SEM implementation are described. In Section 3, we observe the focusing phenomenon submitted to the diffraction limit in two simple homogeneous media. Then, we introduce heterogeneities with split ring shape of subwavelength size. We show that, even if only simple physics is used at the subwavelength scale, once these fine scale structures have been introduced, Helmholtz resonance and frequency band-gaps can be easily observed. From the numerical method perspective, such a modeling is only possible thanks to the very low dispersion and meshing flexibility offered by SEM. Then, in such media, focusing spots with a resolution below the diffraction limit are observed. Finally, in Section 4, we offer some physical interpretations to our observations.

2. Acoustic time reversal equations and numerical implementation

In the present section, we briefly recall the acoustic wave and the time reversal equations. Then an overview of the numerical scheme of the SEM and a description of the time reversal implementation are given. The description and the experiments are done in a 2-D framework but would remain valid in 3-D.