Wave Motion 67 (2016) 121–140

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

A comparison of time-reversal and inverse-source methods for the optimal delivery of wave energy to subsurface targets

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HIGHLIGHTS

- Wave energy focusing to targets in heterogeneous, semi-infinite, elastic media.
- Comparison of time-reversal and inverse-source methods for optimal source design.
- Time-reversal and inverse-source yield similar energy delivery efficiency.

ARTICLE INFO

Article history:
Received 10 May 2016
Received in revised form 24 July 2016
Accepted 26 July 2016
Available online 5 August 2016

Keywords:
Wave energy focusing
Time reversal
Inverse-source modeling
Subsurface targets
Optimal source design

ABSTRACT

Elastic wave stimulation of targets embedded in heterogeneous elastic solids has potential applications in medicine (treatment of tumors, lithotripsy), petroleum engineering, environmental engineering, hydro-geology, etc. Systematic methodologies for designing the wave sources that focus optimally the emitted energy into the target zone are important for ensuring the efficiency of the wave energy delivery process. In this article, we compare two methodologies that could be used for designing wave sources that maximize wave-based stimulation of the target region: a time-reversal (TR) approach and an inverse-source (IS) approach. The former relies on the linearity and reciprocity admitted by the governing physics, and the latter is an optimization-based framework that attempts maximization of a wave motion metric in the target zone. Whereas closed cavity problems are easier to treat, here we concentrate on the challenging problem of wave energy focusing to subsurface geo-formations using motion actuators placed on the ground surface. We formulate the underlying wave propagation problem for two-dimensional, semi-infinite, elastic, heterogeneous domains. We define motion metrics to measure the energy expended by the wave sources, and the kinetic energy delivered to the target formation. We conduct numerical experiments and compare the relative performance of the wave sources designed using the TR and the IS approaches. Both methods have advantages and disadvantages: for remote targets in well-characterized formations IS is preferred; whereas TR is preferable when a source can be embedded in the target. Both methods have similar efficiency.

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http://dx.doi.org/10.1016/j.wavemoti.2016.07.011
0165-2125/

1. Introduction

Stress wave stimulation of a *target zone* in a heterogeneous solid can be a potent, non-invasive recourse for bringing about desired changes in the mechanical behavior of the target. Some key applications of wave stimulation include wave-based enhanced oil recovery (EOR), treatment of tumors and lithotripsy, forced detonation of unexploded ordnances, removal of contaminants from aquifers, and others. Broadly speaking, the success of the aforementioned applications hinges on two types of phenomena: (a) the local behavior, which dictates the intensity of the stimulation required to effect the desired change, and (b) the global behavior, which is concerned with the ability of the wave sources to deliver stimulation of a required magnitude in the presence of application-specific constraints (wave source amplitudes, wave source locations, attenuation, etc.). In general, the local phenomena are governed by microscopic material behavior (pore-scale effects in geophysics, cellular behavior in medical applications, etc.), whereas the global phenomena depend on the macroscopic aspects (the wave path, heterogeneity, constructive/destructive interference, etc.) of wave propagation. This article is concerned with the second aspect (global, or macroscopic) of wave energy delivery. Specifically, we discuss and compare two candidate methodologies, a time-reversal-based approach and an inverse-source approach that can be used for designing wave sources that focus energy to the target zone. In the following, we review the salient features of both methodologies.

The time-reversal method (TR) [1–6] can be used to direct the wave energy towards scatterers or inclusions embedded in an acoustic or an elastic medium. Typical applications of TR include source localization [7,8], crack and defect identification [9,10], land-mine detection [11], wave energy focusing [12], etc. The TR method for focusing wave energy to a target zone consists of two steps. In a first step, the boundary of the (acoustic or elastic) domain of interest, or a part of it, is populated with a receiver array (the time-reversal mirror [2,13]), and a source is placed in the target. The waves emitted by the source are recorded by the receiver array. In a second step, the signals recorded at the receivers are time-reversed, and broadcast back from their respective locations. The signals recorded on the boundary of the medium contain the information about the path the waves traversed. When the mirror-recordings are time-reversed, the emitted waves home in to the location of the original source, i.e., the waves *refocus*. The intensity of the refocus can be ascertained by measuring/computing a motion metric at the target zone. We note that the heterogeneity of the medium does not hinder, but rather enhances the quality of the refocus due to the elongation of the wave path caused by steering and reflection [14]. If one records the waves arriving at the mirror for a sufficiently long time, then waves radiating from the mirror are reflected back to the mirror due to the medium's heterogeneity. Consequently, the record at the mirror has more information compared to what it would have had for a homogeneous medium, and, typically, the quality of the refocus is enhanced [14,15]. Thus, the TR method is a self-adaptive method that accounts for the heterogeneity of the medium [2,13,16]. We remark that an ideal implementation of the TR method requires a perfect mirror, which means that the mirror must cover the entire boundary of the medium. In practice, however, a limited number of sensors, situated only on part of the boundary would be sufficient. Even with this limitation, time-reversal methods work well for focusing if a sufficient time is allowed for the waves to reflect, to change direction, and to reach the boundary where the sensors are placed [3,15,17]. Note that placing wave sources in the target zone may be infeasible or impractical for some applications. In this case, the first TR step can be performed in a numerical simulation.[1] The computed displacement time histories at sensor locations can be time-reversed, and used as source excitations to achieve the desired focusing. Thus, the computational cost of the TR method is (approximately) that of a forward problem solution. Furthermore, if the application at hand allows placement of a wave source in the target zone, then the TR method does not require any *a priori* knowledge of the material and geometric properties of the domain of interest.

Alternatively, wave sources that focus the wave energy to a target subsurface formation can be designed by solving an inverse-source (IS) problem. In previous work in our group [18–21], we formulated an inverse-source problem for wave energy focusing as a constrained minimization problem, where minimization of a suitably defined objective functional is tantamount to the maximization of the wave motion within the target zone, and the equations of the governing physics are side-imposed as constraints. This approach assumes that the material and geometric properties of the domain of interest are known, and aims at finding the optimal source time signals and source locations that maximize the predefined motion metric of the target. The inversion methodology for computing the optimal source parameters faces two key challenges: (a) the solution of the constrained minimization problem is computationally demanding for large elastodynamic systems, and (b) the method assumes *a priori* knowledge of the heterogeneous domain of interest, which is difficult to obtain in practice. However, uncertainties in the material distribution can be addressed, and an assessment of the effect of the uncertainty on the energy delivery efficiency is possible [22].

Thus, both the TR and the IS methodologies can potentially be used to design wave sources that focus wave energy to a targeted subsurface formation. It is unclear though whether the sources designed using the computationally inexpensive TR method will be able to deliver sufficient amount of wave energy to the target zone in a semi-infinite, elastic medium. On the other hand, the optimization-based inversion approach is computationally demanding for large elastodynamic systems. Hence, it is of interest to investigate the relative effectiveness of the two methodologies for geophysical applications. With the notable exception of Levi et al. [23], who reported on TR-based scatterer location recovery in semi-infinite elastic media, the TR method has not been studied for semi-infinite, elastic solids. The key challenges for using time-reversal in semi-infinite, elastic domains include:

[1] Only when the material/geometric description of the domain is known *a priori* with sufficient resolution.