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Finite-difference time-domain method for three-dimensional grid of hexagonal prisms

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HIGHLIGHTS

- FDTD formulation for grid of hexagonal prisms is developed.
- FDTD method for grid of hexagonal prisms is validated with Yee FDTD method.
- Analyses of the numerical anisotropy, dispersion and stability of this FDTD method is made.
- Measurements and theoretical values of numerical anisotropy are compared.

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ABSTRACT

The finite-difference time-domain (FDTD) method was applied in a grid of hexagonal prisms, having as objective to yield less numerical anisotropy of phase velocity than the Yee FDTD method (with hexahedral cells). Comparisons of wave propagation are made between the FDTD method with grid of hexagonal prisms and the Yee FDTD method. The theoretical analyses of the numerical anisotropy, dispersion and stability condition are obtained using the Fourier analysis in the FDTD method with grid of hexagonal prisms. Measurements of numerical anisotropy are also accomplished in this FDTD method, and then ones are compared with the results of the Fourier analysis. As a result, the grid of hexagonal prisms yielded somewhat less numerical anisotropy and dispersion than the Yee grid. Additionally, a simplification in compensation of numerical dispersion in the grid of hexagonal prisms may improve on the accuracy and density of mesh for indoor buildings that are large, mainly in the *xy*-plane.

1. Introduction

Currently, the finite-difference time-domain (FDTD) method [1] can be considering very useful in the analyses of electromagnetic wave propagation in large indoor wireless systems, such as buildings, factories, universities, airports, and so on. It is due to availability of computers more and more powerful and cheap [2,3]. However, the FDTD method has an intrinsic limitation named numerical dispersion [4,5]. The numerical dispersion is the difference between the numerical phase velocity of the electromagnetic wave in two-dimensional (2D) or three-dimensional (3D) grids and the real (physical) phase velocity of this wave in physical media. There is another measurement, which is relating with the numerical dispersion; it is named numerical anisotropy of phase velocity. It means that the numerical phase velocity changes in function of the propagation direction of the wave in 2D or 3D grids. When it is being analyzed the propagation of electromagnetic waves

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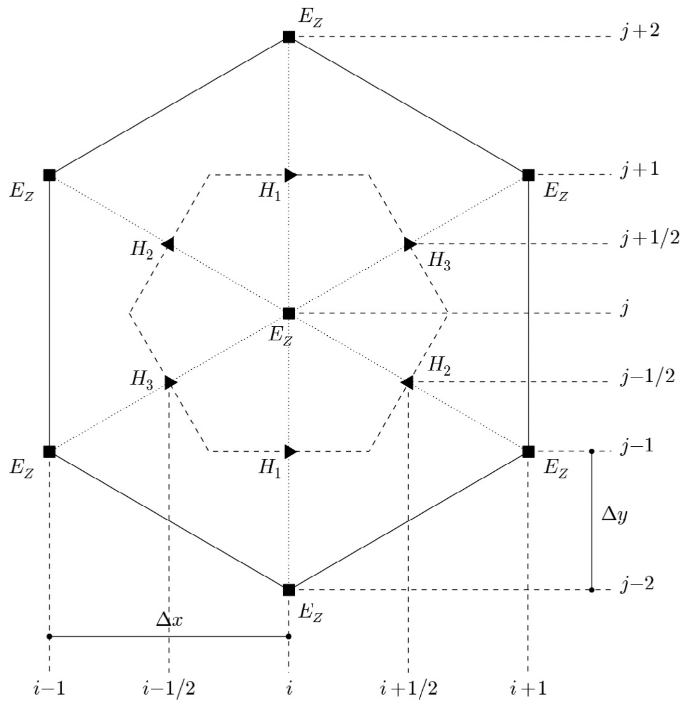

Fig. 1. Primary and secondary cells of the staggered grid of hexagons [9].

in large indoor buildings in terms of wavelengths, it is very important to have low numerical dispersion that reduces phase errors, and therefore increases the accuracy of the simulation [6]. The numerical dispersion is more easily reduced when the maximum numerical anisotropy is also reduced. In this paper, it was developed a novel FDTD method with a grid of hexagonal prisms, which yields much less numerical anisotropy in the *xy*-plane than the Yee FDTD method, and one generates somewhat less numerical anisotropy in *xz*- or *yz*-planes than the Yee method. The results obtained are promising to improve on the accuracy and may also reduce the density of mesh in indoor buildings that are large, mainly in the *xy*-plane.

2. FDTD formulation for two-dimensional hexagonal grid

It is common knowledge, that for the FDTD method, the 2D hexagonal grid has a few hundred times less numerical anisotropy than the 2D rectangular grid of the Yee FDTD method [7,8]. This 2D hexagonal grid is a staggered grid, and it is formed of two different grids. The primary grid consist of big hexagons and the secondary grid consist of small hexagons [7], as shown in Fig. 1.

It can be noted in Fig. 1 that the relation between the side Δb of the small hexagon and the side Δd of the big hexagon is expressed as

$$\Delta b = \frac{\Delta d}{\sqrt{3}}. \tag{1}$$

Here is being used a simpler notation at location of the field components than that of [7]. In this notation, the spatial step size Δx is the distance between mesh points, i and $i+1$, whereas for the spatial step size Δy is the distance between mesh points, j and $j+1$. These spatial step sizes, Δx and Δy, are defined in function of the side Δd of the big hexagon as

$$\Delta x = \Delta d \cdot \cos(30^\circ) = \frac{\sqrt{3}}{2}\Delta d \tag{2}$$

$$\Delta y = \frac{1}{2}\Delta d. \tag{3}$$

The area A_H of the small hexagon is

$$A_H = \frac{3\sqrt{3}}{2}(\Delta b)^2 = \frac{\sqrt{3}}{2}(\Delta d)^2. \tag{4}$$