Wave Motion 63 (2016) 55–67

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Second-harmonic generation in an infinite layered structure with nonlinear spring-type interfaces

Shiro Biwa *, Yosuke Ishii

Department of Aeronautics and Astronautics, Graduate School of Engineering, Kyoto University, Katsura, Nishikyo-ku, Kyoto 615-8540, Japan

HIGHLIGHTS

- The second-harmonic generation in an infinite layered structure due to interfacial nonlinearity is analyzed.
- Using the transfer-matrix method, an explicit analytical expression for the second-harmonic amplitude is obtained.
- The relation between the second-harmonic generation behavior and the band structure of the layered structure is demonstrated.
- Low-frequency approximations of the second-harmonic amplitude are also obtained.

ARTICLE INFO

Article history:
Received 21 September 2015
Accepted 22 January 2016
Available online 4 February 2016

Keywords:
Layered structure
Nonlinear acoustics
Harmonic generation
Perturbation analysis
Spring-type interface
Band structure

ABSTRACT

The second-harmonic generation characteristics in the elastic wave propagation across an infinite layered structure consisting of identical linear elastic layers and nonlinear spring-type interlayer interfaces are analyzed theoretically. The interlayer interfaces are assumed to have identical linear interfacial stiffness but can have different quadratic nonlinearity parameters. Using a perturbation approach and the transfer-matrix method, an explicit analytical expression is derived for the second-harmonic amplitude when the layered structure is impinged by a monochromatic fundamental wave. The analysis shows that the second-harmonic generation behavior depends significantly on the fundamental frequency reflecting the band structure of the layered structure. Two special cases are discussed in order to demonstrate such dependence, i.e., the second-harmonic generation by a single nonlinear interface as well as by multiple consecutive nonlinear interfaces. In particular, when the second-harmonic generation occurs at multiple consecutive nonlinear interfaces, the cumulative growth of the second-harmonic amplitude with distance is only expected in certain frequency ranges where the fundamental as well as the double frequencies belong to the pass bands of the layered structure. Furthermore, a remarkable increase of the second-harmonic amplitude is found near the terminating edge of pass bands. Approximate expressions for the low-frequency range are also obtained, which show the cumulative growth of the second-harmonic amplitude with quadratic frequency dependence.

1. Introduction

Contacting as well as weakly bonded interfaces between solid bodies are often quite different from perfectly bonded interfaces in their behavior when they interact with elastic waves. Such imperfect interfaces are present in various forms in nature as well as in technological products. Acoustic or ultrasonic characterization of imperfect interfaces, e.g., fractures

* Corresponding author. Tel.: +81 75 383 3796.
E-mail address: biwa@kuaero.kyoto-u.ac.jp (S. Biwa).

http://dx.doi.org/10.1016/j.wavemoti.2016.01.004

in rocks [1], machine elements with contacting parts [2,3], kissing bonds and closed cracks in structural members [4–6], has been studied extensively in geophysical exploration and ultrasonic nondestructive testing. When elastic waves impinge on these imperfect interfaces, they exhibit partial reflection and transmission in a frequency-dependent manner. Foregoing studies have revealed that such characteristics can be described by modeling the interfacial region as a spring-type interface characterized by normal and tangential stiffnesses [7–11]. Spring-type interface models have been adopted to analyze the response of closed cracks for different dynamic excitation conditions [12,13]. Waveguide characteristics of imperfect interfaces have been analyzed based on such models and used to characterize them [14–17]. Elastic wave propagation in multilayered structures with spring-type interlayer interfaces has also been analyzed in order to clarify the effect of thin interlaminar resin-rich regions in fiber-reinforced composite laminates [18–23].

Another prominent feature of imperfect interfaces is acoustic nonlinearity [24,25]. In particular, contacting interfaces have been shown to produce significant higher-harmonic components when insonified by narrow-band, high-amplitude waves [26–28]. These features have attracted much attention in the field of ultrasonic nondestructive testing as a sensitive measure to detect closed defects and weak bonding in structural elements. Nonlinear spring-type interface models have been employed to predict the second-harmonic amplitude generated at a contacting interface between solids in reasonable agreement with experimental results [29–34]. In order to examine the second-harmonic generation behavior at a kissing bond interface in an adhesive joint, Yan et al. [35,36] analyzed the one-dimensional wave propagation across an adhesive layer bonded to the adherend using the nonlinear spring-type interface model. They found that the second-harmonic amplitude to be observed is highly dependent on the ratio of the wavelength to the layer thickness due to the band-pass filtering effect of the layer. Ishii and Biwa [37] numerically showed that the second-harmonic generation behavior in multilayered structures with nonlinear spring-type interfaces exhibits remarkably complex frequency dependence. These findings indicate the importance of further examining the second-harmonic generation in multilayered structures due to interfacial nonlinear effects.

Recently, nonlinear elastic wave propagation in multilayered structures has been studied theoretically by different authors. Most of the foregoing studies for this topic consider periodic layered structures consisting of nonlinear elastic materials to explore the amplitude-dependent dispersion and band-gap characteristics [38,39] or the possible occurrence of localized solutions [40–42]. The second-harmonic generation in multilayered structures due to material nonlinearity was analyzed by Yun et al. [43]. It appears, however, that the corresponding issue due to interfacial nonlinearity has rarely been addressed in the existing literature. Better understanding of this issue is not only of academic interest but also expected to give insight into the characterization of weakly bonded structures, e.g., ultrasonic testing of composite laminates with closed delaminations [44,45], acoustic characterization of granular media [46,47], etc.

The aim of this paper is to analyze the second-harmonic generation in the one-dimensional wave propagation across a multilayered structure with nonlinear spring-type interfaces. For a single nonlinear spring-type interface between semi-infinite elastic media, a perturbation analysis for the second-harmonic generation was presented by Biwa et al. [30]. In the present analysis, this foregoing analysis is extended to the case of multiple interfaces in layered structures. To gain fundamental understanding, our attention is focused on an infinite layered structure made of identical linear elastic layers and weakly nonlinear spring-type interfaces with identical linear interfacial stiffness. In Section 2, the formulation is laid down and a perturbation approach is employed to linearize the governing equations. The solution to the fundamental wave propagation is outlined in Section 3 in the frequency domain using the transfer-matrix method [48,49]. The corresponding problem for the second-harmonic component generated by nonlinear interfaces is analyzed in Section 4, and a formal analytical expression for the second-harmonic amplitude in the layered structure is obtained. In Section 5, two special cases are discussed. Namely, the second-harmonic generation characteristics by a single nonlinear interface as well as by multiple consecutive nonlinear interfaces are examined in detail.

2. Formulation

The one-dimensional longitudinal wave propagation across an infinitely extended multilayered structure made of identical linear elastic layers (mass density ρ, wave velocity c and thickness h), which are bonded to each other by spring-type interfaces, is considered as shown in Fig. 1. The wave is assumed to propagate in the direction perpendicular to the layers. The displacement $u(x, t)$ is a function of the position x and time t, and obeys the linear equation of motion in each layer, i.e.,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad X_m < x < X_{m+1}, \tag{1}$$

where $X_m = mh \, (m = 0, \pm 1, \pm 2, \ldots)$ denote the positions of interlayer interfaces. At each interface, the stress $\sigma(x, \ t) = \rho \, c^2 \, \partial u/\partial x$ is continuous while discontinuity is allowed in the displacement. When the spring-type interfaces possess weak quadratic nonlinearity [30], the boundary conditions at $x = X_m$ are given by

$$\rho \, c^2 \frac{\partial u}{\partial x}(X_{m+}, t) = \rho \, c^2 \frac{\partial u}{\partial x}(X_{m-}, t) = K_S \, [\, 1 - \beta_m \, y_m(t)] \, y_m(t), \tag{2}$$

where K_S is the linear interfacial stiffness, assumed to be the same for all interfaces, and β_m is a positive parameter representing the interfacial nonlinearity of the interface at $x = X_m$. Furthermore,

$$y_m(t) = u(X_{m+}, t) - u(X_{m-}, t) \tag{3}$$