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Polarization effects for electromagnetic wave propagation in random media

Liliana Borcea [a], Josselin Garnier [b,*]

[a] Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1043, United States

[b] Laboratoire de Probabilités et Modèles Aléatoires & Laboratoire Jacques-Louis Lions, Université Paris Diderot, 75205 Paris Cedex 13, France

HIGHLIGHTS

- Analysis of Maxwell's equations in random media.
- Quantification of the scattering mean free paths.
- Quantification of the statistical decorrelation of the waves over directions.
- Derivation of the transport equations with polarization for the energy density.
- Quantification of the depolarization.

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ABSTRACT

We study Maxwell's equations in random media with small fluctuations of the electric permittivity. We consider a setup where the waves propagate toward a preferred direction, called range. We decompose the electromagnetic wave field in transverse electric and transverse magnetic plane waves, called modes, with random amplitudes that model cumulative scattering effects in the medium. Their evolution in range is described by a coupled system of stochastic differential equations driven by the random fluctuations of the electric permittivity. We analyze the solution of this system with the Markov limit theorem and obtain a detailed asymptotic characterization of the electromagnetic wave field in the long range limit. In particular, we quantify the loss of coherence of the waves due to scattering by calculating the range scales (scattering mean free paths) on which the mean amplitudes of the modes decay. We also quantify the energy exchange between the modes, and consequently the loss of polarization induced by scattering, by analyzing the Wigner transform (energy density) of the electromagnetic wave field. This analysis involves the derivation of transport equations with polarization. We study in detail these equations and connect the results with the existing literature in radiative transport and paraxial wave propagation.

1. Introduction

Understanding the interaction of electromagnetic waves with complex media through which they propagate is of great importance in applications such as radar imaging and remote sensing [1,2], optical imaging [3], laser beam propagation

* Corresponding author.
E-mail address: garnier@math.univ-paris-diderot.fr (J. Garnier).

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through the atmosphere [4], and communications [5]. As the microstructure (inhomogeneities) of such media cannot be known in detail, we model it as a random field, and thus study Maxwell's equations in random media. The goal is to describe features of the solution, the electromagnetic wave field, which do not depend on the particular realization of the random medium, just on its statistics. Of particular interest in applications are the statistical expectation of the solution, which describes the coherent part of the waves, and the second moments which describe how the waves decorrelate and depolarize, and how energy is transported in the medium.

In most applications the inhomogeneities are weak scatterers, modeled by small fluctuations of the wave speed. They have large cumulative scattering effects at long distances of propagation. Among them are the loss of coherence, manifested mathematically as an exponential decay of the statistical expectation of the wave field, and thus enhancement of the random fluctuations, and wave depolarization. The quantification of these effects depends not only on the amplitude of the fluctuations of the wave speed, but also on the relation between the basic lengths scales: the wavelength, the scale of variations of the medium (correlation length), the spatial support of the source, and the distance of propagation.

Recent mathematical studies of electromagnetic waves in random media are in [6] for layered media, in [7] for waveguides, and in [8] for beam propagation in open environments. They decompose the electromagnetic field in plane wave components, transverse to a preferred direction of propagation called range, and then analyze the evolution of their amplitudes, which are frequency and range dependent random fields. The details and results differ from one study to another, because the geometry and the scaling regimes are different. For example, the decomposition in waveguides leads to a countable set of waves, whereas in open environments, as we consider here, there is a continuum of plane waves. The regime in [8] leads to statistical wave coupling by scattering, however the waves form a narrow cone beam and they retain their initial linear polarization.

In this paper we build on the results in [7,8] to obtain a detailed characterization of polarization effects in random open environments. We consider a medium with random electric permittivity that fluctuates on a scale (correlation length) ℓ that is larger than the wavelength λ by a factor $1/\gamma$, with $\gamma \in (0, 1)$, and the waves propagate over many correlation lengths. By assuming that the support of the source is similar to the correlation length and that the fluctuations of the medium are small and smooth (i.e. their standard deviation α is small and their power spectral density decays fast enough), we identify an interesting regime where the waves propagate along a preferred direction, the range. It is between the paraxial regime studied in [8], where the waves travel in the form of a narrow cone beam, and the radiative transfer regime in [9,10], where the waves travel in all directions. In our regime the waves propagate in a cone whose opening angle is significantly larger than in the paraxial case, but smaller than 180°, so that the backscattered waves can be neglected. The validity of this regime is controlled by the parameter γ and the distance L of propagation. We take $L \gg \ell = \lambda/\gamma$ so that the scattering effects in the medium build up, but to ensure that the waves remain forward propagating, we restrict the distance to $L \sim \alpha^{-2}\ell$.

From the mathematical point of view, the advantage of having a preferred direction of propagation is that we can reduce the analysis of Maxwell's equations to the study of the range evolution of the random amplitudes of the components of the wave field. These amplitudes satisfy a system of stochastic differential equations driven by the fluctuations of the electric permittivity, and can be analyzed in detail using the Markov limit theorem [11,12]. Our main results are:

1. The quantification of the scattering mean free paths, the range scales on which the components of the electromagnetic wave field lose coherence.
2. The quantification of the statistical decorrelation of the waves over directions.
3. The derivation of the transport equations for the energy density, which allows us to quantify the depolarization of the waves and the diffusion of wave energy in direction.
4. The connection of the results to the radiative transport theory with polarization given in [13,9,14], and to the moment equations associated to the random paraxial wave equation given in [15–17] (scalar case) and [8] (electromagnetic case).

The paper is organized as follows: We state up front, in Section 2, the mathematical problem and the main results. The derivation of these results uses the formulation and scaling described in Section 3. The derivation of the wave decomposition is in Section 4. To give an intuitive interpretation of the decomposition we consider first homogeneous media. Then we give the decomposition in random media and derive the system of stochastic differential equations satisfied by the random wave amplitudes. The Markov limit of the solution of this system is obtained in Section 5. We use it in Section 6 to quantify the loss of coherence of the waves. The analysis of the second moments of the amplitudes and the derivation of the transport equations is in Section 7. They are connected to the radiative transport theory in [13,9,10] in Appendix B. We illustrate the results with numerical simulations in statistically isotropic media in Section 8. We give a detailed analysis in the high-frequency limit $\gamma \to 0$ in Section 9 and connect these results to the white-noise paraxial wave equation studied in [8] in Appendix C. We end with a summary in Section 10.

2. Statement of results

We begin in Section 2.1 with Maxwell's equations satisfied by time-harmonic electric and magnetic fields in random media. The scattering regime is defined in Section 2.2, by identifying the important scales and describing their relations. The wave decomposition in transverse electric and magnetic modes is stated in Section 2.3. The characterizations of the first and second moments of the random amplitudes of these modes are our main results, stated in Section 2.4. They are proved in Sections 6 and 7.