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Asymmetrical solitary waves in coupled nonlinear transmission lines

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HIGHLIGHTS

- The asymmetrical pulses developed in coupled nonlinear transmission lines were analyzed on the basis of numerical calculations.
- The closed-form formula of the minimum spatial separation between two pulses interacting repulsively was obtained via reduction theory.
- The repulsive collision between pulses with different polarities could be effectively used to amplify short electrical pulses.

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ABSTRACT

Asymmetrical solitary waves are characterized in symmetrical, capacitively coupled transmission lines, called coupled nonlinear transmission lines (NLTLs), which are periodically loaded with Schottky varactors, in order to amplify short electrical pulses. In this work, the transmission equations of coupled NLTLs are numerically solved, using two symmetrical Korteweg–de Vries (KdV) systems with linear dispersionless coupling, to show that the lines exhibit symmetry breaking in order to support asymmetrical solitary waves. In particular, two interacting solitary waves with different polarities are investigated. The KdV systems show that the polarities of the preceding and the subsequent waves in coupled NLTLs are interchanged through colliding interactions. The process is quantified in the framework of reduction theory, assuming small velocity differences and interaction strength between the two waves. In addition, results show that the leading wave gains amplitude via collision, which means that coupled NLTLs provide good platforms for amplifying short electrical pulses.

1. Introduction

The dynamics of solitary waves in coupled nonlinear systems governed by coupled Korteweg–de Vries (KdV) equations have been investigated at length [1–5]. Espinosa-Cerón et al. [6] recently detailed solitary waves in such a system with linear dispersionless coupling and found that the system supports asymmetrical solitary waves – even if the equations are symmetrical – when their velocity exceeds a threshold, below which only symmetrical solitary waves with identical components are stable. The equations considered in that study are as follows:

$$u_t + 6uu_x + u_{xxx} + \mu v_x = 0, \tag{1}$$

$$v_t + 6vv_x + v_{xxx} + \mu u_x = 0, \tag{2}$$

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where μ is a positive constant. For ease of discussion, a factor 6 is introduced in this paper for nonlinear terms; this factor was not originally used by Espinosa-Cerón et al. Their study found that the solution of symmetrical solitary waves was derived exactly while that of asymmetrical solitary waves was obtained via variational approximations. Because of the asymmetry, if solitary waves of polarity $v > u$ are found to be the solution for a certain velocity, then those with the same fraction but reversed polarity, i.e., $u < v$, are also the solution. Through numerical evaluations, Espinosa-Cerón et al. also found that the collision between asymmetrical solitary waves of opposite polarities had the unique property of reversing the polarity of each wave in the process.

In this paper, we investigate two capacitively coupled nonlinear transmission lines (NLTLs), which are defined as two transmission lines periodically loaded with Schottky diodes by capacitive coupling. Because of the polarity of the diodes, there are two possible configurations. When one of the lines is connected with anodes and the other with cathodes, resonant energy exchange can occur between nonlinear solitary waves carried by the modes, which results in their amplitudes and phases having oscillatory behavior called leapfrogging [7–11]. In contrast, when both lines are connected with anodes, they become symmetrical. When the mutual capacitance is sufficiently small, the lines can be modeled by Eqs. (1) and (2) and may be expected to exhibit symmetry breaking so that asymmetrical solitary waves become stable and several interactions between different types of solitary waves can be observed. This physical observation enhances the impact of the mathematically obtained results. Moreover, future experimental evaluations of asymmetrical solitary waves can establish the validity of both analytically and numerically obtained results. On the other hand, through the effective introduction of asymmetrical solitary waves, the coupled NLTLs can be used to manage traveling electrical pulses. Originally, NLTLs with state-of-the-art Schottky varactors were used in ultrafast electronic circuits [12–14]. The newfound variety of asymmetrical solitary waves and their interactions may extend the potential of NLTL use in ultrafast electronics.

Here, the transmission equations of coupled NLTLs are solved numerically to illustrate the lines symmetry-breaking properties, which are characterized using approximate solutions derived in Ref. [6]. We then investigate the interaction between two asymmetrical solitary waves with different polarities. Consider the case where a faster soliton follows a slower one moving to the common direction, then the faster soliton eventually moves ahead of the slower one via colliding interaction. When we consider the faster and slower solitons have their own identities, it should be considered that the faster one passes the slower one during interaction. On the other hand, we can also consider that the faster (slower) soliton converts to the slower (faster) one by the momentum transfer during interaction. The faster soliton recoils back by the repulsive interaction and loses velocity to become slower than the counterpart soliton. These two ways of interpretation is shown to give indistinguishable interaction duration in the KdV system [15,16]. We assume that the similar equivalence is established in the coupled KdV systems. Moreover, we employ the latter interpretation in what follows unlike Ref. [6]. Numerical calculations show that the separation between spatial peak positions between pulses with different polarities decreases, reaches a minimum, and then increases. In addition, the minimum separation tends to decrease as the velocity fraction between contributing pulses increases. Thus, it is natural for the process to be regarded as the recoiling of trailing pulses caused by the repulsive interaction of leading pulses. We then apply reduction theory to the repulsive pulse interaction to obtain the closed-form expression of the minimum peak separation that qualitatively simulates numerically obtained tendencies. Finally, we discuss the engineering application of asymmetrical solitary waves. The numerical calculations show that leading pulses increase in velocity and gain amplitude by the repulsive interaction they have with trailing pulses of reversed polarity; indeed, the leading pulses amplitude gain increases as the velocity fraction between the interacting pulses increases. This property is effectively used to amplify short electrical pulses.

The rest of the paper is organized as follows. In Section 2, we show the fundamental properties of coupled NLTLs, including the structure, device model, and equations that govern wave propagation; that section also gives a brief review of Ref. [6]. Section 3 discusses the numerical evaluation of collisions between pulses with different polarities, which is validated by the reduction theory for interacting pulses. Finally, Section 4 demonstrates an engineering scheme of pulse amplification.

2. Fundamental properties of coupled NLTLs

Fig. 1 portrays the coupled NLTLs that we investigate. Lines 1 and 2 are weakly coupled via capacitors whose capacitance is C_m. We consider the symmetrical lines, where the line inductance of line 1 is set equal that of line 2; the common-type Schottky diode is used for both lines. In what follows, we call the common line inductance L_0. When reversely biased, the Schottky diode operate as capacitors (called Schottky varactors) whose capacitance depends on the terminal voltage. Both lines 1 and 2 are connected with anodes of loaded Schottky diodes. Moreover, the bias voltages to lines 1 and 2 are commonly set to $-V_b < 0$. Then, the transmission equations of the coupled NLTL are given by

$$L_0 \frac{dI_n}{dt} = V_{n-1} - V_n, \tag{3}$$

$$L_0 \frac{dJ_n}{dt} = W_{n-1} - W_n, \tag{4}$$

$$C_v(V_n - V_b)\frac{dV_n}{dt} + C_m \frac{d}{dt}(V_n - W_n) = I_n - I_{n+1}, \tag{5}$$

$$C_v(W_n - V_b)\frac{dW_n}{dt} + C_m \frac{d}{dt}(W_n - V_n) = J_n - J_{n+1}, \tag{6}$$