Wave Motion 58 (2015) 53–67

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Transformational cloaking from seismic surface waves by micropolar metamaterials with finite couple stiffness

Alexey Khlopotin, Peter Olsson *, Fredrik Larsson

Chalmers University of Technology, Department of Applied Mechanics, SE-412 96 Gothenburg, Sweden

HIGHLIGHTS

- We have studied the possibilities for cloaking of structures using transformational elastodynamics.
- An almost perfect cloak is obtained by letting the size of the structure tend to zero in the fictitious domain.
- Two models for elastodynamic cloaking are considered; the restricted and the unrestricted micropolar medium.
- The more physical unrestricted micropolar medium approaches the restricted case for high couple stiffness.
- The possibility to cloak a cylinder in 2D has been shown, limited by the coefficients in the unrestricted micropolar medium, in numerical simulations.

ARTICLE INFO

Article history:
Received 24 November 2014
Received in revised form 16 June 2015
Accepted 2 July 2015
Available online 8 July 2015

Keywords:
Cloaking
Metamaterials
Micropolar continuum
Transformational elastodynamics
Seismic waves

ABSTRACT

Transformational elastodynamics can be used to protect sensitive structures from harmful waves and vibrations. By designing the material properties in a region around the sensitive structure, a cloak, the incident waves can be redirected as to cause minimal or no harmful response on the pertinent structure. In this paper, we consider such transformational cloaking built up by a suitably designed metamaterial exhibiting micropolar properties. First, a theoretically perfect cloak is obtained by designing the properties of an (unphysical) restricted micropolar material within the surrounding medium. Secondly, we investigate the performance of the cloak under more feasible design criteria, relating to finite elastic parameters. In particular, the behavior of a physically realizable cloak built up by unrestricted micropolar elastic media is investigated. Numerical studies are conducted for the case of buried as well as surface breaking structures in 2D subjected to incident Rayleigh waves pertinent to seismic loading. The studies show how the developed cloaking procedure can be utilized to substantially reduce the response of the structure. In particular, the results indicate the performance of the cloak in relation to constraints on the elastic parameters.

1. Introduction

Ancient civilizations discovered that by arranging the interface between important structures, *e.g.* temples, and the underlying ground, it is possible to achieve some protection against seismic waves, that might otherwise damage the structures. Pliny the Elder describes a successful protective strategy, involving erecting the buildings on foundations with layers of "sheepskins with their fleeces unshorn" and carbon gravel [1]. With this arrangement the buildings could survive many earthquakes with comparatively little damage, by essentially allowing the ground to slip and slide beneath the buildings.

* Corresponding author. Tel.: +46 317723725.
E-mail addresses: alexey.khlopotin@chalmers.se (A. Khlopotin), peter.olsson@chalmers.se (P. Olsson), fredrik.larsson@chalmers.se (F. Larsson).

http://dx.doi.org/10.1016/j.wavemoti.2015.07.002

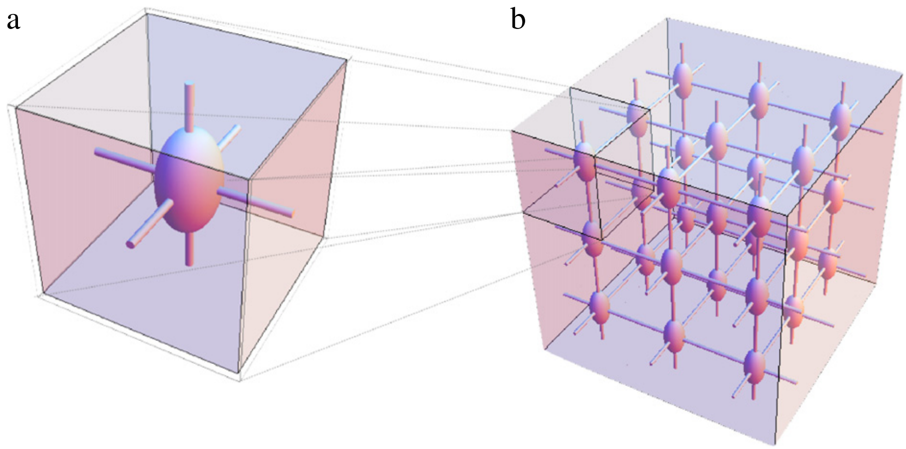

Fig. 1. Schematic sketches of the assumed microstructure of a micropolar material. (a) A typical representative volume element (RVE) on the microscale. A body attached to a connective fabric is embedded in an elastic material. The embedded central body contributes to the microscale moment of inertia tensor, as well as to the macro scale local density. The connective fabric contributes to the microscale couple stiffness tensor. (b) A small part of the micropolar material consisting of a multitude of connected RVEs. Note that a homogenized continuum model is used for the material and no calculations are performed on the microstructural level.

A considerably more modern method to control seismic waves is the use of so-called sonic, or phononic, crystals. The idea is to utilize in an intelligent manner the passband/stopband properties of periodic structures. [2] describes a recent large scale experimental investigation of this type of approach to seismic protection.

Roughly a decade ago, another possible route to earthquake protection was suggested, also relying on the construction of a carefully designed foundation. It started with the discovery of transformational cloaking, initially for electric impedance tomography, by Greenleaf, Lassas, and Uhlmann [3]. Soon to follow were extensions of the approach to electrodynamics [4], elastodynamics [5] and acoustics [6]. By now there is an extensive body of work on elastodynamic and acoustic cloaking. Many approaches, including active cloaking [7,8], as well as the use of pre-stress [9–11], have been proposed. An approximate standard model for cloaking is given in [12].

Early on, it was suggested that cloaking could be utilized to protect structures against seismic surface waves [13], by placing a suitable cloaking layer around the structure to be protected. To put the protecting layer on the surface, rather than bury it beneath, may be motivated by the fact that at least at some distance from the epicenter of the seismic event causing the seismic waves, the harmful waves are to a large extent surface waves, so-called Rayleigh waves, rather than bulk waves. The dominance of surface waves is due to the geometric attenuation in 2D being slower than that in 3D. The type of surface wave cloak described in [13] is an idealized plate, satisfying a wave equation that might be non-trivial to realize in practice. Other cloak models for plates may be found in the literature, *e.g.*, in [14,15].

A perhaps more realistic approach to Rayleigh wave cloaking is to take advantage of the fact that a graded restricted micropolar material may be utilized for cloaking, as described in [16] and generalized in [17]. For a description of the theory of micropolar continua, see, *e.g.*, [18,19]. The presumed microstructure of a micropolar material is sketched in Fig. 1. Note that the figure is only for illustration purposes as a homogenized continuum model is used in this paper. 'Restricted' here indicates that the micropolar material is assumed to have infinite micro stiffness, essentially locking the micro rotation to a constant value, typically zero. As this type of cloak in theory works for any kind of linear elastic bulk wave, it should work also for a Rayleigh surface wave, as this may be considered as a certain superposition of vertically attenuated P and S waves. In the present paper this is verified and checked numerically, using the commercial software COMSOL Multiphysics™.

The practical feasibility of utilizing graded micropolarity for elastodynamic cloaking, hinges to a great extent on how well the condition of infinite micro stiffness (a.k.a. couple stiffness) may be approximated. The problem of infinities required in various cloaking scenarios has been pointed out repeatedly, *cf. e.g.* [6]. But in practice, the infinite may of course often be supplanted by the very large. A case in point is partial cloaking by graded fiber-reinforced composites, where the inextensibility of the fibers may be replaced by a very large stiffness along the fiber direction as compared to the stiffness perpendicular to the fibers [20]. Similarly, it is of interest to see how well a micropolar cloak with high couple stiffness can approximate the (potentially perfect) restricted micropolar cloak. There is also the possibility of mapping the hidden object not to a point (or line) but to a small sphere (or cylinder) so as to avoid infinities due to singular points of the mapping. While this again would make the cloak less than perfect, this drawback might be offset by making it possible to actually manufacture. Similar suggestions for partial cloaking may be found in the literature: in [21], it was proposed to map a ball of small, but finite, radius to the inner boundary, and in [6] and [22] regularization was applied.

Another question, also related to the potential real-world performance of a micropolar cloak, is the extent to which the actual size of the microscale affects the performance of the cloak. While this is of course a matter that properly should be investigated experimentally, it is here suggested that it is possible to get some information on this scale dependence directly from the finite element analysis of the cloak. Contrary to most other contexts, where mesh dependence effects are