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A finite volume approach for the simulation of nonlinear dissipative acoustic wave propagation

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HIGHLIGHTS

- Conservation laws consistent with the Westervelt equation are presented.
- Finite Volume method is implemented and verified for these conservation laws.
- High efficiency is achieved using GPU implementation.

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ABSTRACT

A form of the conservation equations for fluid dynamics is presented, deduced using slightly less restrictive hypothesis than those necessary to obtain the Westervelt equation. This formulation accounts for full wave diffraction, nonlinearity, and thermoviscous dissipative effects. A two-dimensional finite volume method using the Roe linearization was implemented to obtain numerically the solution of the proposed equations. In order to validate the code, two different tests have been performed: one against a special Taylor shock-like analytic solution, the other against published results on a High Intensity Focused Ultrasound (HIFU) system, both with satisfactory results. The code, available under an open source license, is written for parallel execution on a Graphics Processing Unit (GPU), thus improving performance by a factor of over 60 when compared to the standard serial execution finite volume code CLAWPACK 4.6.1, which has been used as reference for the implementation logic as well.

1. Introduction

The Westervelt equation is a classical model for nonlinear acoustic propagation. It was originally obtained in 1963 by P. J. Westervelt [1] and it describes acoustic propagation taking into account the competing effects of nonlinearity and attenuation. This and other nonlinear models for acoustics can be obtained adding hypotheses, commonly in the form of restrictions, to the conservation principles of mass, momentum, and energy. Two other classical nonlinear acoustics models are the KZK and Burgers equations, and both can be obtained adding restrictions to the Westervelt equation: to propagation at small angles from a certain axis (quasi-planar propagation) in the case of the KZK equation, and to propagation strictly along a single axis (planar propagation) for the Burgers equation [2].

With a few notable exceptions, solutions for these nonlinear equations, when known, can only be expressed in non trivial forms, and tools like numerical methods are often required to investigate their nature. Numerical methods, and the

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means required to implement them, have been in continuous development in recent years. Primarily, early publications have been devoted to the description of the more restricted models, like the plane Burgers equation [3], whose exact solution has known analytical expressions [3–5]. Nevertheless, numerical methods in this case have certainly played an important role [6]. After that, the KZK equation became a widely used model for diagnostic and therapeutic medical applications [7], and most of the known solutions have been obtained only by numerical means [8], including some more recent extensions of the model where the restrictions in propagation direction have been partly relaxed [9–12]. Modern medical applications, such as extracorporeal shock wave therapy [13], focus control of high intensity focused ultrasound (HIFU) in heterogeneous media [14], and ultrasound imaging [15], are now demanding more sophisticated solutions to describe systems where the geometric complexity of the nonlinear acoustic field is important. Thus, in recent years, a number of schemes have been produced concerned with implementing methods which are not limited in terms of the propagation direction [16–18,14,15, 19–23], as required in order to solve the Westervelt or Kuznetsov equations, the latter being a model even less restrictive than the Westervelt equation [24]. These numerical methods are sometimes referred to as *full wave methods* [20]. To the best of our knowledge, no general analytic solutions are known for the full wave case either for the Kuznetsov or the Westervelt equations. In the present work we aim to give a full wave numerical solution to a set of conservation laws, obtained using slightly less restrictive hypotheses than those necessary to arrive at the Westervelt equation.

A great number of numerical methods have been used to solve the nonlinear acoustic field, some of them operating over the time domain [19,20,14,21,23], while others involve calculations over the frequency domain [16,17,25,11,18,22]. The numerical method implemented in the present work is a finite volume method, a time domain method. These methods are based on conservation laws, giving them from the start an intrinsic relation to the equations that conform the basis of all acoustic wave models. In the present work the CLAWPACK [26] 4.6.1 serial code, which serves as a standard for finite volume schemes, has been used as a reference for the implementation logic in the presented open source C++/CUDA code, which executes the finite volume method in a GPU graphic card, and notably improves the performance compared to serial schemes. Whereas in the recent literature it is a common practice to use parallelized code for this kind of simulations, this code is mainly run through clusters [17,15,9,14,27,21,28], and GPU execution is just starting to be used [25,29,19,28].

The paper is organized as follows: the relevant equations for this study are described in Section 2; the numerical procedure is described in Section 3; validation tests for the numerical method, and details of their implementation, are given in Section 4; finally, discussion and conclusions are presented in Section 5.

2. Nonlinear acoustic equations

In standard form, the Westervelt equation is given as [2]

$$\nabla^2 p' - \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} + \frac{\delta}{c_0^4}\frac{\partial^3 p'}{\partial t^3} = \frac{-\beta}{\rho_0 c_0^4}\frac{\partial^2 (p')^2}{\partial t^2}, \tag{1}$$

where p' is the acoustic perturbation pressure, ∇^2 is the Laplacian for spatial variables, t is time, c_0 is speed of sound for small signals at an equilibrium state denoted with the zero subscript, β is the coefficient of nonlinearity [30], and δ is sound diffusivity [31].

Since we want to use a finite volume numerical approach, we need to express the Westervelt equation as a system of conservation laws, or more precisely, we need a set of conservation laws as consistent as possible with the Westervelt equation. To begin with, consider the conservation equations for mass and momentum in a compressible fluid, as stated by Hamilton and Morfey [2],

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0, \tag{2}$$

$$\rho\frac{D\mathbf{u}}{Dt} + \nabla p = \mu\nabla^2\mathbf{u} + \left(\mu_B + \frac{1}{3}\mu\right)\nabla(\nabla\cdot\mathbf{u}), \tag{3}$$

where p is the total pressure, ρ is the total mass density, $\mathbf{u}$ is the fluid velocity (null value of the equilibrium state is assumed), μ is the dynamic viscosity, and μ_B is the bulk viscosity. As we have mentioned before, solving these equations, even numerically, requires some assumptions to be made, in this case because the number of variables is greater than the number of relations among them. To keep the mentioned consistency, the hypotheses used here are the same as those used by Hamilton and Morfey [2] to obtain the Westervelt equation, with one exception, which we discuss below. One of these restrictions has to do with the size of the perturbations, p', ρ', and T', considered small and of the same order:

$$\frac{\rho'}{\rho_0}, \frac{p'}{p_0}, \frac{T'}{T_0} = O(\epsilon),$$

where $\epsilon = |u|/c_0$ is the Mach number, T refers to temperature, and p_0, ρ_0, and T_0, are reference values for pressure, density, and temperature, respectively, so that $p = p_0 + p'$, $\rho = \rho_0 + \rho'$, and $T = T_0 + T'$. In addition, the fluid is assumed to be irrotational, we use $\rho\frac{D\mathbf{u}}{Dt} = \frac{\partial\rho\mathbf{u}}{\partial t} + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u})$ to stress the conservative character of Eq. (3), and then we neglect a third