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# Diffraction by a half-plane in uniform translational motion revisited

A. Ciarkowski *

*Warsaw University of Life Sciences, Faculty of Applied Informatics and Mathematics, Nowoursynowska 159, 02-776 Warsaw, Poland*

## HIGHLIGHTS

- Electromagnetic plane wave diffraction by a moving, perfectly conducting half-plane is reconsidered. Relativistic velocities are admitted.
- It is shown that the resulting total field distribution is not simply a moving map of the distribution observed in a stationary case.
- Doppler effect is recognized, and suitable frequencies are found.
- The equation for diffracted wave constant phase surfaces is found.
- The issue of energy transformation in the case of non-parallel shadow boundary and field rays is shortly explained.

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## ABSTRACT

A problem is reconsidered of time harmonic, electromagnetic diffraction by a perfectly conducting half-plane moving in free space with a constant velocity. Similarities and differences between stationary and moving diffraction have been discussed.



## 1. Introduction

The interest in scattering of electromagnetic fields by different objects has a long history. Fundamental principles governing the propagation of geometrical optics field were known yet in middle ages. Later, the effect of diffraction of light at edges of openings in flat screens was analysed and explained in terms of Huygens principle. Solutions to particular problems were naturally approximate. At the turn of 19th and 20th century first exact solutions for simple scatterers were found. In this number was a wedge, and its special case — a half-plane. With developing mathematical methods new tools were used in the study of electromagnetic scattering. The Wiener–Hopf method and asymptotic techniques were here of particular importance. They gave a great momentum to creation of new methods for studying diffraction by scatterers with complex shapes, and allowed for better understanding of physical phenomena accompanying the diffraction.

Most of particular problems dealt with stationary diffraction, i.e. with objects being at rest with regard to the observer and the interacting wave source. However, new technical applications directed the interest towards moving objects. If viewed from the perspective of the Newton space–time transformation, no interesting facts should be expected. From this

* Tel.: +48 22 5937311; fax: +48 22 5937211.
*E-mail address:* adam_ciarkowski@sggw.pl.

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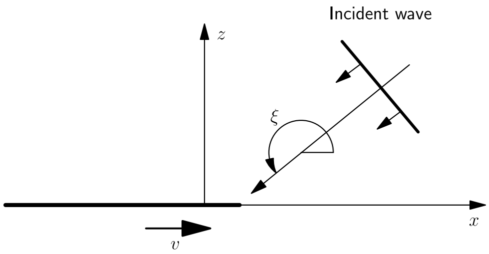


**Fig. 1.** Geometry of the problem.

standpoint the field distribution found for stationary diffraction is "travelling" along with the moving scatterer. However, it is now well known that Newton transformation is only an approximation, admissible for sufficiently small object velocities. The proper approach is Einstein's special theory of relativity, which is founded on Lorentz transformation. That theory is valid for any object velocity, admitting its values varying from zero to relativistic ones. One can expect that if the object velocity is comparable with the velocity of light $c$, at which the electromagnetic field propagates, the scattered field distribution may differ from that in stationary case.

Fortunately, in the construction of exact solutions for scattering by objects in move, already known stationary solutions can be conveniently employed. It can be done with the help of so called Frame Hopping Method which consists in studying diffraction in two frames of reference. The first frame, referred to as "laboratory" one, is the frame, where the incident field is defined and the scattering object is seen as moving with a specific, non-zero velocity. The other frame of reference, referred to as "stationary", is the frame, where the scattering object is at rest. Customarily, all quantities (fields, coordinates) in the stationary frame of reference are primed. They are subject to Lorentz transformation when changing from stationary to laboratory frame of reference, or vice versa. This method was first used by Einstein [1].

Of special interest are the scatterers with edges, where the simplest shape is a half-plane. There is a number of publications in this class of problems [2] through [3], dealing with both harmonic and pulse incident fields. Relativistic half-plane and wedge pulse diffraction was studied in [3,4]. Different formalism in the analysis was used in [2,5]. New differential operators, facilitating field transformations in different inertial frames of reference, were proposed in [6].

In particular, analytic study of time harmonic plane wave diffraction by a perfectly conducting half-plane and physical interpretation of the results was carried out in [7] and [8]. This problem is of special importance, because it is employed in the construction of the solutions of problems involving screen scattering for both harmonic and pulse excitation. The purpose of the present paper is to show already known results in a simpler form, and supplement them with some extension, particularly concerning the Doppler phenomenon. While in [8] two different geometries are considered, we study only one geometry. However the results presented here seem to be simpler in form, as the analysis used here does not involve contour integration in the complex frequency plane.

## 2. Problem formulation

Consider two frames of reference. In the first one the incident field is defined and this is the frame where the scattered field is sought, and the other, wherein the half-plane is at rest. The first frame of reference is referred to as *laboratory* one. In this frame of reference $\{x, y, z, t\}$ the electromagnetic, harmonic plane wave

$$\boldsymbol{E}^i = \boldsymbol{E}_0\, e^{ik(\hat{\boldsymbol{k}}\cdot\boldsymbol{r}-ct)} \qquad c\boldsymbol{B}^i = c\boldsymbol{B}_0\, e^{ik(\hat{\boldsymbol{k}}\cdot\boldsymbol{r}-ct)} \tag{1}$$

is propagating perpendicularly to the $y$ axis and under the angle $\xi$, $\pi < \xi < 2\pi$, with respect to the $x$ axis. The unit vector $\hat{\boldsymbol{k}}$ in the direction of propagation and the radius vector are given by

$$\hat{\boldsymbol{k}} = [\cos\xi, 0, \sin\xi], \qquad \boldsymbol{r} = [x, y, z]. \tag{2}$$

Here, $k$, $c$ and $t$ stand for wave number, velocity of light and time, respectively. The incident wave is scattered by a perfectly conducting half-plane, which is moving along the $x$ direction with a constant velocity $\boldsymbol{v}$ (see Fig. 1). This velocity may take values from zero to relativistic ones. It is assumed that the edge of the half-plane reaches $x = 0$ at the moment $t = 0$.

The other frame of reference $\{x', y', z', t'\}$ is chosen, wherein the half-plane is at rest. This frame is referred as *stationary* one, and the half-plane is described by $x' \le 0$, $z' = 0$ in it.

Our goal is to find the resulting total electromagnetic field in the laboratory frame. Since the incident field is independent of $y$ coordinate and the half-plane is uniform along that direction, the problem here analysed is 2D.

## 3. The solving method

The idea of solving this problem is due to Einstein. The incident field is first Lorentz transformed from the laboratory to stationary frame of reference, and the total field resulting from diffraction of the transformed incident field by the stationary half-plane, is found. This field is then Lorentz transformed back to the laboratory frame of reference. In the literature the method is referred to as Frame Hopping Method.

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