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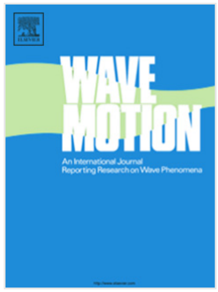

Causality analysis of the fractional state and wave equations based on fractional Laplacians

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Causality analysis of the fractional state and wave equations based on fractional Laplacians

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Abstract

This paper is concerned with the investigation of the state and wave equations based on fractional Laplacians which have some popularity in Photoacoustics tomography (PAT). It is shown that such state equations are *nonlocal*, more precisely, a *local* density variation causes an *instant global* pressure variation and a *local* pressure variation can only be caused by an *instant global* density variation. This is in contrast to all frequency dependent dissipative state equations known to the author. Moreover, it is shown that the Green function G of the respective wave equation does not have a *finite* wave front speed. To obtain a fractional wave equation with a finite wave front speed, we propose a local fractional state equation that is similar to the original fractional state equation. We note that our approach is readily applied to many state and wave equations based on fractional space derivatives. Finally, we present numerical simulations that clearly visualize the noncausal behavior of the underlying space fractional state equations.

1 Introduction

The modeling of wave and diffusion equations with *fractional Laplacians* is very popular in science but causality problems are caused due to the nonlocality of fractional Laplacians. In contrast to diffusion, a fractional wave is based on a state equation which determines its causal behavior. That is to say, if *causal* fractional waves exist, then there must exist fractional state