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Effects of the local resonance in bending on the longitudinal vibrations of reticulated beams

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HIGHLIGHTS

- The local resonance is due to the bending/compression stiffness contrast in beams.
- It is evidenced analytically by homogenization and confirmed numerically.
- The macroscopic description contains a frequency dependent effective mass.
- A structure can have the same macroscopic mode shape at different frequencies.
- The vibrations are not transmitted when the effective mass is negative.

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ABSTRACT

This work investigates the dynamic behaviour of reticulated beams obtained by repeating a unit cell made up of interconnected beams or plates forming an unbraced frame. As beams are much stiffer in tension–compression than in bending, the longitudinal modes of such structures (governed by tension–compression at the macroscopic scale) can appear in the same frequency range as the bending modes of the elements. The condition of scale separation being respected for compression, the homogenization method of periodic discrete media is used to rigorously derive the macroscopic behaviour at the leading order. In the absence of bending resonance, the longitudinal vibrations of the structure are described at the macroscopic scale by the usual equation for beams in tension–compression. When there is resonance, the form of the equation is unchanged but the real mass of the structure is replaced by an effective mass which depends on the frequency. This induces an abnormal response in the neighbourhood of the natural frequencies of the resonating elements. This paper focuses on the consequences on the modal properties and the transfer function of the reticulated structure. The same macroscopic mode shape can be associated with several natural frequencies of the structure (but the deformation of the elements at the local scale is different). Moreover the vibrations are not transmitted when the effective mass is negative. These phenomena are first evidenced theoretically and then illustrated with numerical simulations.

1. Introduction

Locally resonant materials or metamaterials are a class of composite materials with a high stiffness contrast between the constituents. The propagation of waves in the stiff component with a wavelength much greater than the heterogeneity

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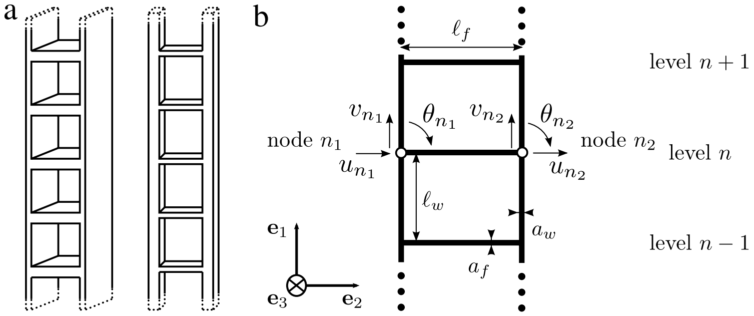

Fig. 1. (a) Examples of studied structures; (b) notation.

size can then induce the resonance of the soft component. This phenomenon which differs from diffraction leads to unusual effective properties investigated in the pioneering work of Auriault and Bonnet in 1985 [1] (see also [2]) and observed experimentally in [3,4]. In particular, the effective density is different from the real density and depends on the frequency. The description of such composites at the macroscopic scale is a generalization of the Newtonian mechanics. This question is frequently addressed with mass–spring models (such as the Maxwell–Rayleigh model cited in [5]) which are difficult to realize in practice. The stratified composite studied by Auriault and Bonnet and the reticulated structure considered in this paper are more realistic systems.

Indeed, in [6,7], it was shown that reticulated materials with only one constituent can also behave as locally resonant materials. In that case, the stiffness contrast comes from the geometry of the microstructure. Reticulated materials are made up of interconnected beams or plates. Examples include materials of millimetric size such as foams, plants, bones, of metric size such as the sandwich panels, stiffened plates and truss beams used in aerospace and marine structures, of decametric size such as buildings. Since beams and plates are much stiffer in tension–compression than in bending, the propagation of compressional waves with a long wavelength and the local bending modes of the elements can occur in the same frequency range. The local resonance in bending of a reticulated material is used in [8] to attenuate vibrations over desired frequency ranges.

In this paper, we investigate the consequences of the local resonance in bending on the dynamic behaviour of periodic frame structures. Instead of considering wave propagation as in [6,7], emphasis is put on the modification of the features of the longitudinal modes. For the first modes of a structure with a sufficiently large number of periods (or cells), deformations occur on a length scale much greater than the size of a period. Therefore the homogenization method of periodic discrete media (HPDM method) can be used to obtain a macroscopic description. This method, elaborated by Caillerie [9] has been extended by a systematic use of scaling based on dimensional analysis [10,11] and applied to situations with local resonance [6,7]. Its main advantages are that the macroscopic behaviour is derived rigorously from the properties of the basic frame and that it provides an analytic formulation which enables to understand the role of each parameter. This method has already given interesting results on the transverse dynamics of frame structures [11].

The framework of the study is described in Section 2 and the details of the HPDM method are given in Appendix B. Section 3 presents the two possible macroscopic behaviours: without and with local resonance. In Section 4, the consequences of the local resonance on the free and forced vibrations are analysed. These results are confirmed by finite element simulations. Finally, Section 5 discusses the potential applications of this work. The differences between the idealized reticulated structures and real buildings are examined and the important points for the design of new structures with prescribed properties are highlighted. Note that the demonstrations of some results about the harmonic vibration of Euler–Bernoulli beams used in this article are gathered in Appendix A.

2. Framework of the study

2.1. Studied structures and kinematic descriptors

The studied structures are constituted by a pile of a large number N of identical unbraced frames called cells and made of a floor supported by two walls (see Fig. 1). The walls and the floors are beams or plates which behave as Euler–Bernoulli beams in out-of-plane motion. They are linked by perfectly stiff and massless nodes. The characteristics of the floors ($j = f$) and the walls ($j = w$) are: ℓ_j length, a_j thickness, h depth in the direction $\mathbf{e}_3$, $A_j = a_j h$ cross-section area, $I_j = a_j^3 h/12$ second moment of area in the direction $\mathbf{e}_3$, ρ_j density, E_j elastic modulus.

This paper deals with the harmonic vibrations of the structure at the unknown circular frequency ω of the longitudinal modes. Therefore, every variable can be written in the following way: $X(t) = \Re(X\,e^{i\omega t})$ where t is the time. Since the study is conducted within the framework of the small strain theory and the linear elasticity, the time dependence can be simplified and will be systematically omitted.

As explained in Appendix B, the HPDM method begins with the discretization of the dynamic balance. The study of the momentum balance of the whole structure is exactly replaced by the study of the momentum balance of the nodes. Since

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