Available online at www.sciencedirect.com
ScienceDirect
International Journal of Engineering Science 46 (2008) 374–390
International Journal of Engineering Science
www.elsevier.com/locate/ijengsci

Fracture analysis of a magnetoelectroelastic solid with a penny-shaped crack by considering the effects of the opening crack interior

Xian-Ci Zhong [a], Xian-Fang Li [a,b,*]

[a] *Institute of Mechanics and Sensor Technology, School of Civil Engineering and Architecture, Central South University, Changsha, Hunan 410083, China*
[b] *Centre for Advanced Materials Technology (CAMT), School of Aerospace, Mechanical and Mechatronic Engineering, University of Sydney, Sydney, NSW 2006, Australia*

Received 30 March 2007; accepted 12 November 2007
Available online 26 December 2007

Abstract

A magnetoelectroelastic analysis for a penny-shaped crack embedded in an infinite piezoelectromagnetic material is made. Taking into account the fact that electric and magnetic fields can permeate through the opening crack, the electric and magnetic boundary conditions at the crack surfaces are assumed to be semi-permeable, or depend nonlinearly on the crack opening displacement. For the case of a circular crack normal to the poling direction, the associated mixed boundary value problem is reduced to solving dual integral equations by applying the Hankel transform technique. An entire magnetoelectroelastic field is obtained in simple and explicit form. Numerical results for a cracked $BaTiO_3$–$CoFe_2O_4$ material reveal the dependence of the electric displacement and magnetic induction at the crack surfaces with applied mechanical loading. The influences of applied electric and magnetic loadings on normalized fracture parameters are illustrated graphically for a vacuum circular crack. The impermeable and permeable cracks can be treated as two limiting cases of the present.

Keywords: Magnetoelectroelastic solid; Penny-shaped crack; Electric and magnetic boundary conditions; Full field

1. Introduction

Magnetoelectroelastic solids made of piezoelectric and piezomagnetic materials are a class of new smart structures, possessing potential wide applications. Recently, due to the brittleness and low strength of such materials, the failure analysis of magnetoelectroelastic solids with defects, particularly cracks, has attracted

* Corresponding author. Address: Institute of Mechanics and Sensor Technology, School of Civil Engineering and Architecture, Central South University, Changsha, Hunan 410083, China.
E-mail address: xfli@mail.csu.edu.cn (X.-F. Li).

0020-7225/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijengsci.2007.11.005

increasing attention. For two-dimensional problems in an infinite magnetoelectroelastic solid, Liu et al. [14] have studied the problem of an elliptical cavity or a crack in an anisotropic magnetoelectroelastic material using Green's functions. Wang and Mai [23] have considered a mode-III crack problem and obtained the path-independent integral near the crack tip. The multi-crack interaction problems in magnetoelectroelastic materials have been studied by Gao et al. [5,6], Tian and Gabbert [20] and Tian and Rajapakse [21], respectively. The interface crack problems in two magnetoelectroelastic solids have been investigated by Gao et al. [5], Soh and Liu [19], Wang and Mai [24] and Li and Kardomateas [9], respectively. Wu et al. [27] have dealt with the problem of stress concentration for a hyperbolic notch in a magnetoelectroelastic material. The dynamic problems for a cracked magnetoelectroelastic medium have been solved by Li [10], Zhou et al. [31] and Feng et al. [4], respectively. A crack running along the interface of two dissimilar magnetoelectroelastic materials has been formulated by Zhong and Li [29]. An infinite magnetoelectroelastic solid containing a spheroidal inclusion has been analyzed by Hou and Leung [8]. Three-dimensional magnetoelectroelastic analysis has been investigated by Chen et al. [1], Niraula and Wang [16], Zhao et al. [28] and Tian and Rajapakse [22], etc.

For the above-mentioned works, two kinds of ideal crack-face magnetoelectrical boundary conditions are used, that is, (a) electrically and magnetically permeable, i.e. $\phi^+ = \phi^-$, $D^+ = D^-$, $\varphi^+ = \varphi^-$, $B^+ = B^-$ (e.g., [5,23,24]), or (b) electrically and magnetically impermeable, i.e. $D^+ = D^- = 0$, $B^+ = B^- = 0$ (e.g., 10,4,22]). The former case simply treats the crack surfaces in both electrical and magnetic contact, although the crack is opened. Evidently, this assumption neglects the presence of the crack interior such as vacuum or air. As for the latter case, the electric displacement and magnetic induction are simply assumed to vanish, equivalent to the assumption that the crack interior is not permeable for electric and magnetic fields. This is also contradictory to a realistic case, since a realistic crack is full of certain medium (usually air or vacuum) with nonvanishing electric permittivity and magnetic permeability, which should not be neglected although they may be small enough. Wang and Mai [25] analyzed the influence of the material constants of the crack interior on field intensity factors. In order to simulate more natural magnetoelectrical crack-face boundary conditions, similar to the assumption in [7] for a cracked piezoelectric material, we propose the following magnetoelectrical crack-face boundary conditions

$$D^c = -\varepsilon^c \frac{\Delta\phi}{\Delta u_z}, \qquad B^c = -\mu^c \frac{\Delta\varphi}{\Delta u_z}, \tag{1}$$

where $\varepsilon^c = \varepsilon_r \varepsilon_0$ ($\varepsilon_0 = 8.85 \times 10^{-12} F/m$) and $\mu^c = \mu_r \mu_0$ ($\mu_0 = 1.26 \times 10^{-6} Ns^2/C^2$); $\Delta\phi$ and $\Delta\varphi$ are the jumps of electric and magnetic potentials across the crack. Here ε_r and μ_r are relative constants depending on the permeability of the electric and magnetic fields inside crack, respectively. Generally, in solving an associated boundary value problem in elasticity theorem, boundary conditions are commonly given based on a state prior to deformation. If still adopting such a treatment, clearly the assumption of the permeable crack can be derived for a mathematical crack of no thickness. Just as pointed out, when a crack opens under combined loadings, the crack interior cannot be neglected. Thus, the nonlinear relations in (1) can be understood as a result posterior to deformation [13].

Furthermore, when a crack is filled with an ideal vacuum crack, $\varepsilon_r = \mu_r = 1$. Specially, four limiting cases are: (i) a completely impermeable crack, i.e., $\varepsilon_r = \mu_r = 0$; (ii) a completely permeable one, i.e., $\varepsilon_r \to \infty$ and $\mu_r \to \infty$; (iii) an electrically permeable and magnetically impermeable one, i.e., $\varepsilon_r \to \infty$ and $\mu_r = 0$; (iv) an electrically impermeable and magnetically permeable one, i.e., $\varepsilon_r = 0$, and $\mu_r \to \infty$. Note that previous works are mainly focused on either completely impermeable or completely permeable cracks. Recently, using the so-called semi-permeable conditions (1), the mechanical, magnetic, and electric behaviours of a two-dimensional piezoelectromagnetic material with a crack of finite length has been dealt with in [30].

This paper is concerned with a three-dimensional magnetoelectroelastic solid with a penny-shaped crack subjected to the crack-face conditions (1). The Hankel transform technique is employed to solve this problem. A complete magnetoelectromechanical field in the entire space can be derived. Obtained results reveal that the stress intensity factors are consistent with their counterparts in a purely elastic medium with a circular crack. In contrast, other intensity factors are dependent on applied stress as well as the crack interior properties. The influences of the crack interior properties on fracture parameters are presented graphically and discussed.