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Electro-elastic interaction between a piezoelectric screw dislocation and collinear rigid lines

B.J. Chen *, D.W. Shu, Z.M. Xiao

School of Mechanical and Aerospace Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore

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Abstract

Electro-elastic stress investigation on the interaction between a piezoelectric screw dislocation and collinear rigid lines under anti-plane mechanical and in-plane electrical loading is carried out. The lines are considered, respectively, as dielectrics or conductors. The screw dislocation is subjected to a line charge and a line force at the core. Closed-form analytical solutions are derived by means of complex variable method. Explicit expressions for the field variables, the singularity of the field variables at the line tip and the force on the dislocation are obtained for a single rigid line.

Keywords: Piezoelectric; Screw dislocation; Rigid line; Interaction

1. Introduction

Piezoelectric materials are widely used in the device applications such as sensors and actuators. When subjected to mechanical and electric loads, these piezoelectric materials can fail prematurely due to defects produced during their manufacturing process. It is therefore important to know how the defects, such as cracks, dislocations and inhomogeneities, disturb the field variables and how the stress concentration arises due to the existence of the defects.

For a Mode III crack problem in a homogenous piezoelectric material, Pak [1] found that the crack growth could be either enhanced or retarded depending on the magnitude, the direction, and the type of the applied mechanical and electrical loads. Especially for certain ratios of the applied electrical load to mechanical load, crack arrestment can be observed. Pak [2] obtained the closed-form solutions for a screw dislocation in a piezoelectric solid subjected to external loads and calculated the generalized Peach–Koehler forces acting on the screw dislocation.

* Corresponding author. Fax: +65 67936763.
E-mail address: mbjchen@ntu.edu.sg (B.J. Chen).

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In the case of a circular inclusion in a homogenous piezoelectric material, Pak [3] analyzed and showed that in modeling cavities, imposing an impermeable boundary condition is a good approximation provided that the piezoelectric material has high dielectric constant and strong electro-elastic coupling. Stress and electric field concentrations are also studied. It is shown that a high electric field can be induced in the inclusion under a mechanical load when the matrix and the inclusion are poled in the opposite directions. Later, Meguid and Zhong [4] provided a theoretical treatment for the elliptical inhomogeneity problem in piezoelectric materials under remote non-uniform anti-plane shear and in-plane electric field using the complex variable method.

There are also a number of research works on the interaction problems among cracks, dislocations and inclusions in piezoelectric solids published in open literature for the anti-plane deformation case. Meguid and Deng [5,6] studied the interaction between a screw dislocation and an elliptical inhomogeneity in piezoelectric media. Lee et al. [7] examined the interaction between a semi-infinite crack and a screw dislocation under anti-plane mechanical and in-plane electrical loading. Chen et al. [8] investigated the case of a wedge shaped crack interacting with a screw dislocation. Zhang et al. [9] derived the electrical and mechanical fields induced by a screw dislocation near an electrically insulating elliptical cavity in a piezoelectric material through the image dislocation approach by considering the electric field inside the cavity and found that the difference in the electric boundary conditions leads to great differences in the image force acting on the dislocation, in the intensity factors and in the J integral for crack propagation induced by the dislocation. Li and Weng [10] investigated the problem of a finite crack in a strip of functionally graded piezoelectric material and shown that an increase in the gradient of the material properties can reduce the magnitude of the stress intensity factor.

When a flat inclusion is much harder that the matrix, it is reasonable to be considered as a rigid line. Shi [11] investigated the collinear rigid lines under anti-plane deformation and in-plane electric field in piezoelectric media. Chen et al. [12] studied the interaction between a semi-infinite rigid line and a piezoelectric screw dislocation. Liu and Fang [13] studied the interaction problem between a piezoelectric screw dislocation and circular interfacial rigid lines. The rigid line problem is a counterpart of the conventional crack problem, and thus sometimes called as anti-crack problem.

It is therefore the purpose of this work to study the problem of how a piezoelectric screw dislocation interacting with collinear rigid lines under anti-plane deformation and in-plane electric field.

2. Formulation of the problem

The physical problem considered is shown in Fig. 1. A charged screw dislocation is located at the point $z_d(r_d, \theta_d)$ near some rigid lines embedded in an infinite piezoelectric medium under remote anti-plane strain deformation and in-plane electric field. The rigid lines are assumed to be collinearly located along the x-axis of a Cartesian coordinate system xyz. The dislocation is assumed to be straight and infinitely long in the z-direction, suffering a finite discontinuity in the displacement b_z and electric potential b_φ across the slip plane. The dislocation has a line force p and a line charge q along its core.

In a linear piezoelectric medium, the governing field equations and constitutive relations at constant temperature can be written as

$$\sigma_{ij,j} = 0, \tag{2.1a}$$
$$D_{i,i} = 0, \tag{2.1b}$$

$$\sigma_{ij} = c_{ijkl}u_{k,l} - e_{kij}E_k, \tag{2.2a}$$
$$D_i = e_{ikl}u_{k,l} + \varepsilon_{ik}E_k, \tag{2.2b}$$

where σ_{ij}, u_i, D_i and E_i are stress, displacement, electric displacement and electric fields, respectively. c_{ijkl}, e_{kij} and ε_{ij} are the corresponding elastic, piezoelectric and dielectric constants which satisfy the following relations

$$c_{ijkl} = c_{klij} = c_{ijlk} = c_{jikl}, \quad e_{kij} = e_{kji}, \quad \varepsilon_{ik} = \varepsilon_{ki}. \tag{2.3}$$

As for the current an anti-plane problem, the anti-plane displacement w is coupled with the in-plane electric field E_x and E_y, those variables are independent of the longitudinal coordinate z, such that