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Some peculiarities of the asymmetric Graetz problem

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Abstract

A numerical simulation and an analysis of the steady state forced convection heat transfer with plane laminar flow confined by two parallel plates that are kept at constant but different temperatures are presented. We name this heat transfer configuration shortly the *asymmetric Graetz problem*. The essential features of the asymmetric in comparison to the symmetric Graetz problem are the reversal of the heat flux and the jump of the Nusselt number from positive to negative region at the plate having the temperature closer to the fluid inlet temperature. These phenomena occur at different axial positions, which depend on the thermal asymmetry and the fluid inlet conditions. The numerical results agree excellently with an analytical solution obtained in terms of Kummer confluent hypergeometric function and Hermite polynomials.

Keywords: Graetz problem; Thermal asymmetry; Heat transfer

1. Introduction and scope of the paper

The interest in single-phase forced convection with laminar duct flows originated from an article by Graetz [1] in 1883 and reached – in engineering sense – a culmination in the late 1950s of the last century. Graetz assumed a constant wall temperature and a developed velocity field in a circular tube and arrived at a solution of this heat transfer situation that was later called the *symmetric Graetz problem*. Sellars et al. [2] solved it in the convenient form. Meanwhile, various formulations of the classical Graetz problem have been examined and the results summarised in several monographs, e.g., by Petukhov [3], Shah and London [4], and Kakaç et al. [5]. These sources are recommended for original contributions and details, like boundary and flow conditions, or correlations for heat transfer and pressure drop. The most recent review of heat transfer

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Nomenclature

A	constant
b	halve distance of plates
D	constant
d_h	hydraulic diameter, Eq. (4)
E	constant
k	thermal conductivity
q_W	wall heat flux
Nu	Nusselt number, Eq. (16)
Pr	Prandtl number
Re	Reynolds number, Eq. (4)
u	local velocity in x-direction
w	local velocity in z-direction
$\bar{w}$	average fluid velocity
x, y, z	coordinates
X	dimensionless coordinate, Eq. (4)
Z	dimensionless coordinate, Eq. (4)
Z_*	dimensionless coordinate, Eq. (10)
α	heat transfer coefficient
κ	thermal diffusivity
ϑ	temperature
θ	dimensionless temperature, Eq. (9)
$\bar{\theta}$	average of θ
Θ	dimensionless temperature, Eq. (A.2)
$\overline{\Theta}$	average of Θ
φ	eigenfunction, Eq. (A.8)
ν	kinematic viscosity

Sub- and superscripts

W	at wall, plate
h	hydraulic
IN	at inlet
n	current index
φ', φ''	derivatives of φ with respect to X
1	wall (plate) 1
2	wall (plate) 2

correlations in this area was provided by Muzychka and Yovanovich [6]. At present there are hardly contributions in the literature dealing with the Graetz problem in its classical formulation.

As can be taken from the monographs [3–5], numerous papers are devoted to plane asymmetric Graetz problem when one plate is isothermal and the other one adiabatical. Both developed and developing flows were considered. The case of hydrodynamically developed flow at prescribed heat fluxes was extensively studied by Petukhov [3], who observed both negative and positive Nusselt numbers, depending on the ratio of specified heat fluxes. Also Shah and London [4] reported on the similar results. Such conditions, however, are seldom encountered in common practice, if any. Developing flows – particularly from thermal point of view – play a much more important role in practice, and the question as to whether the Nusselt number can change the sign in the region of the thermal development possesses not only an academical interest.

Modern publications in this area are devoted to specific formulations of the Graetz problem. For instance, Weigand et al. [7,8] considered the heat transfer in a fluid flowing in a parallel plate channel and a concentric