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Cross-property connections for fiber reinforced piezoelectric materials with anisotropic constituents

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Abstract

The paper addresses the problem of the connection between effective elastic stiffnesses, piezoelectric coefficients and dielectric permeabilities of a fiber reinforced piezoelectric composite with both phases (the matrix and the fibers) being transversely-isotropic. These connections allow one to predict the entire set of macroscopic elastic stiffnesses through one or two measurements of dielectric permeability. The solutions for square and hexagonal arrays of fibers and for randomly located parallel fibers are constructed and compared. The analytical results show very good agreement with available experimental data. As a side result, it is shown that the mutual positions of inhomogeneities produce only a minor effect and that applicability of the non-interaction approximation is much wider than expected.

Keywords: Fiber reinforced composites; Effective moduli; Cross-property connections; Asymptotic homogenization; Piezoelectric material

1. Introduction

Micromechanical analysis of heterogeneous materials is usually directed towards (a) explicit expressions of the effective properties in terms of the microstructural parameters; (b) obtaining bounds on the effective properties; and (c) obtaining relations between different effective properties. While the first two objectives can be considered as traditional ones (we refer to review papers of Willis [38] and Markov [23]), the third one represents a less developed area.

It appears that the problem of cross-property connection was first stated by Bristow [6] who interrelated macroscopic Young's modulus and electric conductivity of a material containing randomly oriented cracks. Hill [18] constructed relations between elastic constants of a fiber reinforced composite – he showed that there

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are only three independent effective elastic stiffnesses instead of five. Levin [21] interrelated the effective bulk modulus and the effective thermal expansion coefficient of a two phase isotropic composite. His results were extended to anisotropic materials by Rosen and Hashin [28].

The problem was restated in a different form in the works of Milton [25] and Berryman and Milton [3] where the cross-property bounds – i.e. bounds for elastic constants in terms of conductivities instead of microstructural parameters – were established. Based on the results of Gibiansky [8,9], Gibiansky and Torquato [12–15] advanced this problem by constructing *realizable* cross-property bounds for two-dimensional elastic constants and three-dimensional bulk modulus of isotropic composites. The advantage of this approach is that it produces universal bounds, in the sense that they are independent of microgeometries. The mathematical aspects of cross-property bounds were discussed in the books of [26,37].

A different approach to the cross-property connection was proposed in papers of [19,32,31]. In contrast with the *exact bounds,* these papers construct *approximate explicit cross-property connections* between elastic and conductive properties of inhomogeneous materials in the framework of non-interaction approximation.

Two sets of cross-property connections for *piezoelectric* materials were developed. Benveniste and Dvorak [2] extended results of Hill [18] and showed that there are five independent effective constants instead of 10 in a transversely-isotropic fiber reinforced piezoelectric composites. This number was further reduced by Schulgasser [30] who derived one more connection on the basis of the general result of Milgrom and Shtrikman [24]. Dunn [11] generalized results of Levin [21] and Rosen and Hashin [28] to the case of thermoelectroelastic moduli.

In the present paper, we extend the cross-property connections to fiber reinforced materials with *anisotropic* (transversely-isotropic) constituents and establish connections between the changes (due to the presence of fibers) in piezoelectric constants, elastic stiffnesses, and dielectric permeabilities. The paper further develops results of Sevostianov et al. [34], where elasticity/conductivity connections are established for fiber reinforced composites with anisotropic constituents. We consider three types of fiber reinforcement – square and hexagonal periodic arrays and randomly located fibers. For the first two cases solution is based on the results of Bravo-Castillero et al. [4], Guinovart-Diaz et al. [17], Rodriguez-Ramos et al [27], and Sabina et al. [29] obtained by asymptotic homogenization technique. The basic results for a composite with randomly located fibers are given by Sevostianov et al. [33] in the frameworks of the non-interaction approximation (NIA) and Kanaun–Levin effective field scheme (KLS) [20]. We compare theoretical cross-property connections with the experimental measurements of [7] in Section 5.

2. Basic results

In this section, we briefly outline the background results for our analysis: governing equations for piezoelectric materials, Benveniste–Dvorak's and Schulgasser's universal relations, and solution of Eshelby's problem for piezoelectrics.

2.1. Governing equations

The linear constitutive equations for a piezoelectric material have the form:

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon_{kl} - e_{ijm}E_m \\ D_i &= e_{ijkl}\varepsilon_{kl} + \zeta_{im}E_m \end{aligned} \tag{2.1}$$

where ε_{ij} and σ_{ij} are strains and stresses, D_i are the electric displacements (components of the electric induction vector) and E_i are components of the electric field vector. These quantities are connected by fourth-rank tensor of elastic stiffnesses C_{ijkl} for a fixed electric field, third-rank tensor of piezoelectric constants e_{ijm} and second-rank tensor of dielectric permeabilities ζ_{im} for a fixed strain.

For a transversely-isotropic material with the x_3 axis being the axis of symmetry, these relations can be rewritten in the following form using the two-index notations for material constants: