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The concept of a balance law for a cracked elastic body and the configurational force and moment at the crack tip

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Abstract

In this work, the derivation of the configurational equations for a cracked elastic body by postulating balance laws, is studied. To this end, a proper kinematics is proposed, according to which the evolution of the crack in the material configuration and the physical motion (deformation) are separated. A rigorous localization procedure provides the local equations in material space holding in the smooth part of the body, the corresponding jump conditions along the crack, as well as the configurational force and moment at the crack tip. The results are discussed in connection with the corresponding ones of fracture mechanics and some new interpretations are proposed.

Keywords: Configurational forces; Balance laws; Crack; Energy release rate; Eshelby stress tensor

1. Introduction

The objective of this paper is a systematic development of the global balance laws for an elastic fractured body and the investigation of their consequences. According to a standard procedure of continuum mechanics, all the relevant equations can be produced by postulating global balance laws for mass, momentum, angular momentum and energy. This procedure is now enlarged to account for the configurational fields, as well.

In this enlargement, two main points must be taken into account. At first, the involved fields are not continuous across the crack, particularly, they may have a singularity at the crack tip and second, the underlying kinematics is more complicated due to the presence of the propagating crack [1].

Reports to equations, which can be viewed as balance laws for a fractured body, appear in the works of Maugin, Gurtin and their colleagues [2–6]. Formerly, Golebiewska Herrmann [7] using Noether's Theorem has provided balance laws in material space by examining their relationships with *J*- and *L*-integrals. Recently, an interesting paper has been published by Steinmann [8] providing a unified view of balance laws for

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elastostatics, in both the physical and material space. The present work aims at the formulation of balance laws for elastodynamics in the presence of a crack, such that the possible singularities of the involved fields to be taken into account during the localization process.

This unified view can shed light on the relationship between the configurational fields at the crack tip and the energy release rates. For instance, starting from the pseudomomentum and energy equations, one can establish a connection between the energy release rate and the configurational force at the crack tip, which is directly related to the J-integral [4,5]. One of our goal in this paper is to explore an analogous relationship starting from the material angular momentum and energy equations. In this case, it is expected that there is some connection between the configurational moment at the crack tip and the rotational energy release rate.

Golebiewska Herrmann and Herrmann [9] studied the rotation of a crack and computed the corresponding rotational energy release rate. Eischen and Herrmann [10] tried to connect the conservation (and balance) laws with the energy release rates and the J-, L- and M-integrals. In these works, a straight stationary crack is considered as well as the rotational energy release rate that emerges by a virtual rotation of the crack around its center. Although this is a very successful and meaningful manipulation (in the sense that the associated conservation law is coming from the invariance of the action functional under the group of rotations), it cannot be related to a real situation of a propagating crack.

Looking for a more physical interpretation, the propagation of a crack along a curve of arbitrary curvature is considered in such a way that the linear and the angular velocity of the crack tip to be inserted into the picture. The balance laws are postulated and from the localization process the configurational fields at the crack tip naturally arise. Finally, an attempt is made to correlate these quantities with the energy release rates and the classical J- and L-integrals of fracture mechanics.

Although the crack propagation in a deformable body is a dissipative phenomenon, in this paper no mention is made to the second law of thermodynamics and to the subsequent discussion about the constitutive relations. The analysis is based on the finite two-dimensional elasticity and on the framework of the so-called configurational (or material) mechanics. We refer to the monographs of Maugin, Gurtin, Kinzler and Herrmann [4,2,11] for an extended exposition of the main ideas of configurational mechanics.

In Section 2, some preliminaries concerning the proper kinematics for a cracked elastic body are presented. In Section 3, an abstract balance law is postulated, the conditions under which it is meaningful are examined and its consequences are extracted. The application of this procedure to the configurational fields, related to the problem under study, is made in Section 4. Finally, in Section 5, the obtained results are used to derive the relations between the energy release rates and the configurational fields at the crack tip.

2. Kinematics and some preliminaries

Let $\mathscr{B}_R$ be the reference configuration of a two-dimensional elastic body containing a crack. This is described by a smooth, non-intersecting curve C_R with the one end point lying on the boundary of the body and the other one being the crack tip, $\mathbf{Z}_0$ (Fig. 1). We consider that the crack evolves, not necessarily in

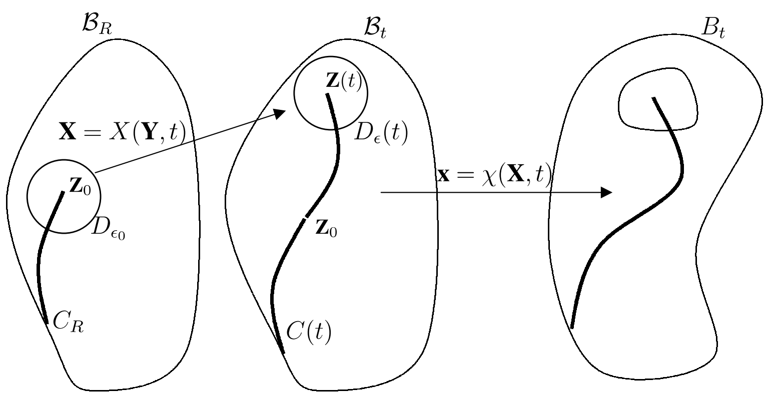

Fig. 1. The configurations.