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Expansion formulae for wave structure interaction problems with applications in hydroelasticity

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Abstract

Alternate derivations of the expansion formulae for wave structure interaction problems are obtained in case of water of infinite depth and utilized to analyze the hydroelastic behavior of large floating structures. Considering the boundary value problem associated with Laplace equation having higher order boundary condition on the horizontal boundary and a Dirichlet type boundary condition on the vertical boundary in a quarter plane, Fourier sine transform is applied in the horizontal direction to convert the problem to a Sturm–Liouville type boundary value problem associated with non-homogeneous ordinary differential equation (ODE) in the transformed variable. Finally, inverting the transformed functions and applying the regularity criterion of the transformed function, the required expansion formula is derived. The expansion formula thus derived is extended to deal with similar boundary value problems having Neumann type boundary condition. The expansion formulae are applied to (i) analyze oblique scattering of flexural gravity waves by an articulated floating elastic plate and (ii) study the effect of compression on the oblique scattering of flexural gravity waves by a line discontinuity in a large floating ice sheet in water of infinite depth, which find applications in marine technology and arctic engineering, respectively. The present derivations of the expansion formulae are very simple and straightforward and can be easily used to study a large class of problems in the area of fluids and structures in mathematical physics and engineering.

Keywords: Expansion formulae; Velocity potential; Fourier sine transform; Floating elastic plate; Flexural gravity wave; Hydroelasticity

1. Introduction

The term hydroelasticity first appeared in the technical literature in the spring of 1958 (in the first symposium of Naval Structural Mechanics sponsored by Office of the Naval Research and Stanford University), is undoubtedly coined by analogy to aeroelasticity to denote its naval counterparts [1]. Although, the subject started keeping floating vessels in mind, the related principles and methodologies are now being applied to a wide variety of marine structures [2]. The main advantages of hydroelasticity theory compared to rigid body

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analysis are the physically more accurate idealization of the fluid structure interaction system and consequently the more rigorous analysis by which dynamic responses, such as the stresses and bending moments, in waves are obtained. In recent decades, this subject has gained considerable importance to analyze wave interaction with very large floating structures (VLFS) and mobile offshore base (MOB) for utilization of ocean space for various humanitarian and military operations [3].

To study wave-induced structural responses of large floating structures in regular waves, very often structural and hydrodynamic analysis are performed separately. In general, finite element method or mode expansion method is used for structural analysis and the panel method is used for fluid analysis. In the mode expansion method, Newman [4] proposed to employ different orthogonal polynomials to represent the corresponding mode expansions for different edge conditions and it is often difficult to find the fittest modal functions for various types of edge conditions. In addition, in the aforementioned procedure, it is often difficult to obtain quantitative information on the reflection and transmission coefficients when a regular wave is incident on the plate. On the other hand, Ohkusu and Namba [5] proposed a direct method to analyze the response of a thin horizontal floating elastic plate by combining the kinematic and dynamic free surface conditions in order to obtain the free surface condition on the plate covered region and then solved the problem by boundary integral equation method. The free surface condition obtained by Ohkusu and Namba [5] is similar to the one used in the analysis of wave interaction with floating ice sheet [6]. It may be noted that the resulting free surface condition in the plate covered region gives important information regarding the wave numbers associated with the flexural gravity waves. Various aspects of the wave interaction with large floating elastic plate are analyzed by Namba and Ohkusu [7], Sturova [8], Kashiwagi [3], Sahoo et al. [9], Hermans [10], Watanabe et al. [11] and the literature cited therein. A detail review on the recent developments of hydroelasticity theories in marine technology are available in Chen et al. [12].

On the other hand, the most popular model for analysis of ocean wave interaction with floating ice sheet is based on the assumption of the ice sheet as an elastic plate, which was originally visualized by Greenhill [6]. In addition, floating ice surface is being used as airport runway since 1957/1958 [13]. Schulkes et al. [14] investigated the flexural gravity wave pattern excited on a floating ice plate by taking into account the effect of (i) compressive stress in the plane of the plate, (ii) uniform flow in the underlying water and (iii) stratification of the underlying water and made various observations on the flexural gravity wave pattern. Detail developments on waves propagating below floating ice sheet were reviewed by Squire et al. [15]. Milinazzo et al. [16] analyzed the steady response of the floating ice sheet to the uniform motion of a rectangular load. The analysis shows a significant dependence of the amplitude of the ice displacement on the aspect ratio of the load. Further, it has been observed that no steady state solution is possible when the load speed equals the minimum speed of the flexural gravity waves propagate below the ice sheet. The above two types of problems have a common feature that Euler–Bernoulli beam theory is being used for structural analysis while surface gravity wave theory is being used for the wave motion in the fluid region under the assumption that the thickness of the structure is negligible. Fox and Squire [17] studied the scattering of ocean waves by large floating ice sheet in two-dimensions based on Timoshenko–Mindlin equation for the structural analysis in order to accommodate comparatively thick ice sheet. Using a similar modelling as developed by Fox and Squire [17], Balmforth and Craster [18] analyzed the oblique wave scattering by floating ice sheet of finite thickness based on Wiener–Hopf technique and obtained the solution of the problem in terms of various Wiener–Hopf integrals. Chung and Fox [19] simplified the analysis of Balmforth and Craster [18] by making the required computation straightforward. Porter and Porter [20] investigated the scattering of flexural gravity waves by an ice sheet of variable thickness over an undulating bed topography using the mild-slope approximation.

One of the major difficulties in dealing with problems related to wave interaction with floating elastic structures of the type mentioned above is the existence of higher order boundary conditions for a Laplace (or Helmholtz) as the governing equation in an unbounded domain. Lawrie and Abrahams [21] obtained an orthogonal relation for a general class of boundary value problems of Helmholtz type partial differential equation with higher-order boundary condition in a semi-infinite strip domain. Chakrabarti et al. [22] utilized the regularity criteria of the Fourier cosine transform to obtain a Fourier-type expansion formula for the flexural gravity wavemaker problem in infinite water depth. However, the unknown coefficients in the expansion