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International Journal of Engineering Science 45 (2007) 41–54

International Journal of Engineering Science

www.elsevier.com/locate/ijengsci

On the dynamics of non-linear viscoelastic solids with material moduli that depend upon pressure

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Received 20 July 2006; accepted 21 July 2006
Available online 20 February 2007

Abstract

We investigate the dynamic response, of a generalization of an incompressible Kelvin–Voigt viscoelastic solid, whose viscosity depends on the pressure. Bodies with pressure-dependent material moduli have relevance to numerous technologically significant problems in geomechanics, the mechanics of granular media and powder compaction. We obtain analytical results for creep and recovery phenomena as well as solutions to the propagation of waves in such bodies. We are able to obtain explicit exact solutions that clearly illustrate the marked difference in the response of bodies with pressure-dependent material moduli as opposed to their counterparts whose moduli do not depend on the pressure. We also show that the governing equations for such materials can change type, and that their solutions exhibit singularities and localization.
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Keywords: Non-linear viscoelasticty; Material moduli; Pressure dependence; Shearing

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1. Introduction

Recently, there has been considerable interest in describing the behavior of incompressible fluids whose material moduli depend upon the pressure, in virtue of its technological relevance. The aim of the present paper is to investigate the counterpart of such research with respect to solids as such studies have equally important scientific and technological ramifications. Relevance to such problems can be found in geomechanics, glaciology, processing of granular materials, to name a few areas.

In geological applications, especially when one is concerned with the earth's mantle, the significant range of mechanical loads that are encountered require one to take into account the fact that the viscosity of the material depends on the pressure. For example while modeling the rheology of the layered lithosphere it is important to take into consideration the slip rate variations, since the fault slip history depends on how the strain is

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doi:10.1016/j.ijengsci.2006.07.015

accumulated during the history of the deformation. The observation that accelerated or decelerated motion of the fault may be sustained over the period of many years even when the external force is removed [9] could possibly be explained by the fact that the viscosity depends on the pressure in virtue of the following reasoning: It is possible that in the lower strata where the pressure is higher, strain is released slower than in higher strata, where the pressure and therefore the viscosity are lower. This can have significant effect on the movement of the strata. It is equally important to take the effect of pressure on the properties of lava that emanate from the earth core at very high pressures to the surface wherein the pressures are significantly lower, if one is to predict their motion with desired accuracy. Yet another important example in geomechanics is the phenomenon of liquefaction wherein one observes a catastrophic collapse of geological structures due to increase in pore pressures. A reasonable model for the phenomenon has to take into consideration the dependence of the material moduli that characterize the body to depend on the pressure. Similarly, in granular materials, especially while sintering them at high pressure it is critical to take the pressure dependence of the material moduli into account.

The pressure dependence of viscosity is a well documented experimental fact in polymer mechanics. Many constitutive models have been empirically adapted to take this fact into account, but usually these models are presented only within a one-dimensional framework. Recently Hausnerova et al. [8] consider a generalization of the Carreau–Yasuda model wherein the viscosity depends on the pressure, to study injection moulding problems. However, their model is essentially a generalized fluid model and they are not concerned with general issues concerning the mechanics of solids with pressure-dependent material moduli, which is the specific concern of this paper. Before we get into a discussion of the problem of interest, it would be useful to briefly discuss the recent advances with regard to the understanding of the mechanics of fluids with pressure dependent viscosities.

That the viscosity of liquids could depend upon the pressure was known to the pioneers of the field. Stokes [20] is in fact very careful to delineate the special class of flows, those in channels and pipes at moderate pressures, when viscosity could be assumed a constant. There is also a considerable amount of literature even prior to 1930 concerning the variation of viscosity with pressure (see the book by Bridgman [3] on the physics of high pressures for a detailed discussion and [1,5,7,12,13] for later experimental results).

Recently, Rajagopal and coworkers have studied in detail an interesting modification to the classical Navier–Stokes model wherein the fluid is assumed to be incompressible with the viscosity depending on the pressure; which is equivalent to the viscosity depending on the Lagrange multiplier that enforces the constraint (see [14,10] and the survey paper by Rajagopal [16]).

Indeed, there are many applications; elasto-hydrodynamics being one, where the fluid can be modeled as an incompressible fluid with a viscosity that depends on the pressure (see [21]). The justification for such an assumption stems from the fact that while the density changes by merely a few percent, the pressure can change significantly and the viscosity can change by several orders of magnitude. For instance the percent change in the density when the pressure changes from 2 to 3 GPa is approximately 3.5% (see the recent experiments of Bair and Kottke [2] and the discussion in [17]). In fact, experiments unequivocally show that the changes in density due to changes in pressure, at high pressures, are indeed negligible.

In continuum mechanics it is not standard to encounter theories, for incompressible materials, where constitutive functions may depend on the pressure and we can only ascribe popular prejudice with respect to such theories amongst theoreticians for their shying away from studying them. To understand the reason for such a state of affairs we need merely look at the theory for incompressible materials. As in classical mechanics, also in continuum mechanics, it is not sufficient to specify an internal constraint only from the kinematical point of view, it is also necessary to introduce a dynamical characterization. This is usually done by appealing to what is usually referred to in continuum mechanics as the “principle of determinism for simple materials” subject to simple constraints. This assumption states that the Cauchy stress for such materials is determined (at any time) by the history of the deformation gradient only to within a stress that does no work in any motion satisfying the constraints. This is quite a restrictive point of view (that this is indeed so even within the context of rigid body mechanics was recognized in the seminal work of Gauss [6]). Indeed, to think that all the internal constraints have to satisfy this principle is akin thinking that in classical mechanics the only possible constraint is the ideal one. Rajagopal and Saccomandi [18] have shown that allowing a dependence on the pressure for the viscosity is a fundamental step towards generalizing the theory of internal constraint along a pathway that it is

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