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International Journal of Engineering Science 44 (2006) 1346–1352
International Journal of Engineering Science
www.elsevier.com/locate/ijengsci

The energetics and the stability of axially moving strings undergoing planar motion

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Received 21 March 2006; received in revised form 7 August 2006; accepted 9 August 2006
Available online 27 October 2006

Abstract

Free coupled planer vibration of an axially moving string is investigated from the point of view of energetics. The time-rate of the total mechanical energy associated with the vibration is calculated for axially accelerating strings with ends moving in a prescribed way. The result shows that the energy is not conserved for a string moving in a constant axial speed and constrained by two fixed ends. For such a string, it is proved that there exists a conserved quantity that remains a constant during the coupled planar vibration. An approximate conserved quantity is derived from the conserved quantity in the neighborhood of the straight equilibrium configuration. The approximate conserved quantity is applied to verify the Lyapunov stability of the straight equilibrium configuration.

Keywords: Axially moving string; Planar vibration; Energetics; Conserved quantity; Stability

1. Introduction

Axially moving strings can represent many engineering devices such as power transmission belts, plastic films, magnetic tapes, paper sheets and textile fibers [1,2]. The total mechanical energy associated with axially moving string is not constant even if the strings travel between two supports, while there do exist alternative quantities that are conserved. Both energy and conserved quantities are potentially useful for stability analysis and controller design. Besides, they can be used to develop and to validate numerical algorithms.

Energetics and conserved quantities are of considerable interest in the study of axially moving strings. Chubachi first discussed periodicity of the energy transfer in an axially moving string [3]. Miranker analyzed energetics of an axially moving string, and derived an expression for the time rate of change of the string energy [4]. Roos et al. considered heat transfer to the moving string in the energy analysis [5]. Wickert and Mote pointed out that Miranker's expression represents the local rate of change only because it neglected the energy flux at the supports, and they presented the temporal variation of the total energy related to the

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0020-7225/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.ijengsci.2006.08.007

local rate of change through the application of the one-dimensional transport theorem [6]. Renshaw examined the change of the total mechanical energy of two prototypical winching problems, which provided strikingly different examples of energy flux at a fixed orifice of an axially moving system [7]. Lee and Mote presented a generalized treatment of energetics of axially moving strings [8]. Renshaw et al. defined a conserved quantity for axially moving strings [9]. Zhu and Ni investigated energetics of axially moving strings with arbitrarily varying length, and applied the results in the stability analysis [10]. Chen and Zu analyzed the energetics and defined a conserved quantity based on Mote's nonlinear model of axially moving strings [11]. Their results are used to check numerical schemes [12–14]. Chen and Zhao proposed a conserved quantity for axially moving Kirchhoff nonlinear strings and applied it to the stability analyses [15]. However, all aforementioned investigations on energetics and conserved quantities of axially moving strings have only been limited to transverse vibration, in which longitudinal motion is assumed to be uncoupled and thus neglectable. In addition, energetics was studied only for strings moving with a constant axial speed. To address the lack of research in this aspect, the author investigates energetics and conserved quantity for axially accelerating strings undergoing coupled planar nonlinear vibration.

In this work, the energetics is investigated for axially accelerating strings. The time-rate of the energy change is calculated for ends moving in a given way. A physical interpretation of the time-rate is presented. A conserved quantity is constructed for axially moving strings under fixed boundary conditions. The conserved quantity leads to an approximate conserved quantity that can be applied to prove the Lyapunov stability of the straight equilibrium configuration of an axially moving string on the condition that the axial speed is less than the critical speed.

2. Governing equation

Consider a uniform axially moving string of linear density ρ, cross-sectional area A, and initial tension P_0. The string travels at the uniform transport speed $\gamma(t)$, which is a prescribed function of time t, between two boundaries separated by distance L. Assume that the deformation of the string is confined to the vertical plane. The string is subjected to no external loads. A mixed Eulerian–Lagrangian description [16] is adopted. The distance from the left boundary is measured by fixed axial coordinate x. The in-plane vibration of the string is specified by the longitudinal displacement $u(x,t)$ related to coordinate translating at speed $\gamma(t)$ and the transverse displacement $v(x,t)$ related to a spatial frame. Here $u(x,t)$ and $v(x,t)$ describe respectively the longitudinal and transverse displacements of the string element instantaneously located at axial spatial coordinate x and time t, though different material elements occupy that position at different times. The physical system is shown in Fig. 1.

For an elastic string with Young's modulus E, the governing equation for coupled planar motion is [17]

$$\begin{aligned}
&\rho\left(u_{,tt}+\dot{\gamma}(1+u_{,x})+2\gamma u_{,xt}+\gamma^2 u_{,xx}\right)-\frac{\partial}{\partial x}\left[\frac{(P_0+AE\varepsilon)(1+u_{,x})}{\sqrt{(1+u_{,x})^2+v_{,x}^2}}\right]=0,\\
&\rho(v_{,tt}+\dot{\gamma}v_{,x}+2\gamma v_{,xt}+\gamma^2 v_{,xx})-\frac{\partial}{\partial x}\left[\frac{(P_0+AE\varepsilon)v_{,x}}{\sqrt{(1+u_{,x})^2+v_{,x}^2}}\right]=0.
\end{aligned}\tag{1}$$

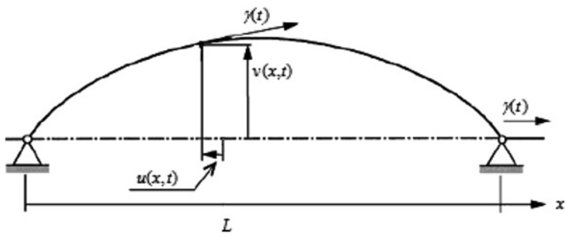

Fig. 1. An axially accelerating string.