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ScienceDirect
International Journal of Engineering Science 45 (2007) 467–485
International Journal of Engineering Science
www.elsevier.com/locate/ijengsci

Analytical solution for functionally graded magneto-electro-elastic plane beams

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Received 17 August 2006; accepted 15 December 2006

Abstract

This paper investigates the plane stress problem of generally anisotropic magneto-electro-elastic beams with the coefficients of elastic compliance, piezoelectricity, dielectric impermeability, piezomagnetism, magnetoelectricity, and magnetic permeability being arbitrary functions of the thickness coordinate. Firstly, partial differential equations governing stress function, electric displacement function and magnetic induction function are derived for plane problems of anisotropic functionally graded magneto-electro-elastic materials. Secondly, these functions are assumed in forms of polynomials in the longitudinal coordinate and can be acquired through a successive integral approach. The analytical expressions of axial force, bending moment, shear force, average electric displacement, average magnetic induction, displacements, electric potential and magnetic potential are then deduced. Thirdly, problems of functionally graded magneto-electro-elastic plane beams are considered, with integral constants being completely determinable from boundary conditions. A series of analytical solutions are thus obtained, including the solutions for beams under tension and pure bending, for cantilever beams subjected to shear force applied at the free end, and for cantilever beams subjected to uniform load. These solutions can be easily degenerated into the solutions for homogenous anisotropic magneto-electro-elastic beams. Finally, a numerical example is presented to show the application of the proposed method.

Keywords: Magneto-electro-elastic material; Functionally graded; Plane beam; Anisotropic

1. Introduction

Composites made of magneto-electro-elastic (MEE) material can exhibit the magnetoelectric coupling that is not present in the single-phase piezoelectric or piezomagnetic material [1–3]. Thus, the magneto-electro-mechanical coupling problems, which intensively interest scientists and engineers, have been becoming a

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0020-7225/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.ijengsci.2007.03.005

hot topic, and numerous papers have been published. For example, Liu et al. [4] studied the Green's functions for an infinite two-dimensional (2D) anisotropic MEE medium containing an elliptical cavity. Pan [5] derived the 3D exact solutions for simply supported, linearly anisotropic multi-layered MEE rectangular plates under static loads. The analysis was later extended to free vibration by Pan and Heyliger [6]. 3D Green's functions in anisotropic MEE full space, half space, and bimaterials were obtained by Pan [7] based on the extended Stroh formalism along with 2D Fourier transforms. Hou et al. [8] analyzed the elliptical Hertzian contact of transversely isotropic MEE bodies by virtue of the general solution that was expressed in terms of harmonic functions. Wang and Zhong [9] presented a general solution of 3D equilibrium problem of spherically isotropic MEE media and obtained the exact and compact form solutions for a cone subjected to concentrated force, concentrated couple, a point charge and a point electric current at its apex as well as for some other cases. Wang and Shen [10] put forward a general solution of 3D problems of transversely isotropic MEE media expressed by five potential functions. Chen et al. [11] formulated a 3D general solution in terms of harmonic functions for transversely isotropic magneto-electro-thermo-elastic media and studied the problem of a penny-shaped crack subjected to a combination of mechanical, electrical, magnetic and thermal loads. By virtue of the operator theory, Ding and Jiang [12,13] derived the closed-form 2D and 3D general solutions in the case of distinct eigenvalues. With the 2D general solution, Jiang and Ding [14] obtained Green's functions for point forces, point charge and point current acting in the interior of an infinite MEE half-plane. They also utilized the general solution to study problems of beams under tension and pure bending, cantilever beam under point force, point charge or point current at the free end and cantilever beam subjected to uniform load [15].

Functionally graded materials (FGMs) have found wide applications in engineering. In contrast with traditional materials, FGMs represent a class of composites with inhomogeneous distribution of different components in the material. The devices made of FGMs can adapt to much bad environments, have high reliability, and play essential roles in most advanced integrated systems of vibration control and health monitoring. Chen et al. [16,17] investigated the bending and free vibration problems of a transversely isotropic functionally graded MEE (FGMEE) plate using a particular state-space formulation along with the approximate laminate model. Pan and Han [18] gave an exact solution for functionally graded and layered MEE plates with all material constants varying in the same exponential way along the thickness direction. Bhangale and Ganesan [19] carried out static analysis of linearly anisotropic FGMEE plates using a semi-analytical finite element method. Bhangale and Ganesan [20,21] reported the finite element investigations on free vibration of FGMEE plates and cylindrical shells. To the authors' knowledge, no literature about static analysis of FGMEE beams with different kinds of support has been published yet.

This paper investigated the plane stress problem of general anisotropic FGMEE plane beams with coefficients of elastic compliance, piezoelectricity, dielectric impermeability, piezomagnetism, magnetic permeability, and magneto-electricity varying arbitrarily along the thickness direction. The basic formulations for FGMEE plane problem are presented in Section 2. In Section 3, the stress function, electric displacement function and magnetic induction function are premised and determined with an integral approach. The expressions for internal forces, average electric displacement, average magnetic induction, displacements, electric potential, and magnetic potential are presented in Section 4. Solutions to three typical problems, of which the pure elastic counterparts are very classical in elasticity, are presented in Section 5, which can be degenerated into the solutions for homogeneous beams. A particular example is considered in Section 6 with numerical results presented.

2. Basic formulations

The basic formulations of the plane stress problem for MEE materials include the equilibrium equations, the geometric equations and the constitutive equations as follows.

The equilibrium equations:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xz}}{\partial z}=0,\quad \frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\sigma_z}{\partial z}=0, \tag{1}$$