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Similarity solutions of a MHD boundary-layer flow past a continuous moving surface

Emad H. Aly [a,*], M. Benlahsen [b], M. Guedda [c]

[a] *Department of Civil and Building Engineering, Loughborough University, Loughborough LE11 3TU, United Kingdom*
[b] *LPMC, Université de Picardie Jules Verne, Faculté des Sciences, 33, Rue Saint-Leu 80039 Amiens, France*
[c] *LAMFA, CNRS UMR 6140, Université de Picardie Jules Verne, Faculté de Mathématiques et d'Informatique, 33, Rue Saint-Leu 80039 Amiens, France*

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Abstract

This note deals with a theoretical and numerical analysis of multiple similarity solutions of the two-dimensional MHD boundary-layer flow over a permeable surface, with a power law stretching velocity, in the presence of a magnetic field B applied normally to the surface. We have taken the free stream velocity to vary as x^m, where x is the coordinate along the plate measured from the leading edge and m is a constant. The magnetic field B is assumed to be proportional to $x^{\frac{m-1}{2}}$. The problem depends on the power law exponent and the magnetic parameter M or the Stewart number. It is shown, under certain circumstance, that the problem has an infinite number of solutions.

Keywords: Boundary-layer; Porous media; Similarity solutions; Existence; Non-existence; Multiple solutions

1. Introduction

In the present paper, we will examine the similarity solutions of the two-dimensional boundary-layer flow over a semi-infinite surface, in the presence of a transverse magnetic field $B(x)$, given by the special form $B(x) = B_0 x^{\frac{m-1}{2}},\ B_0 \neq 0$, where x is the coordinate along the plate measured from the leading edge and m is a constant. The surface velocity and the free stream velocity are given by $u_s(x) = u_w x^m$ and $u_e(x) = u_\infty x^m$, respectively. Fig. 1 schematically illustrates the mathematical model of the problem under investigation. The governing boundary-layer differential equation is given by [40,41,45],

$$f''' + \frac{1+m}{2} ff'' + m(1 - f'^2) + M(1 - f') = 0, \tag{1}$$

where the primes denote differentiations with respect to the similarity variable $\eta \in (0, \infty)$, f denotes the similar stream function and its derivative, after suitable normalization, represents the velocity parallel to

* Corresponding author. Tel.: +44 0 1509 223214; fax: +44 0 1509 223923.
E-mail address: E.H.Aly2@Lboro.ac.uk (E.H. Aly).

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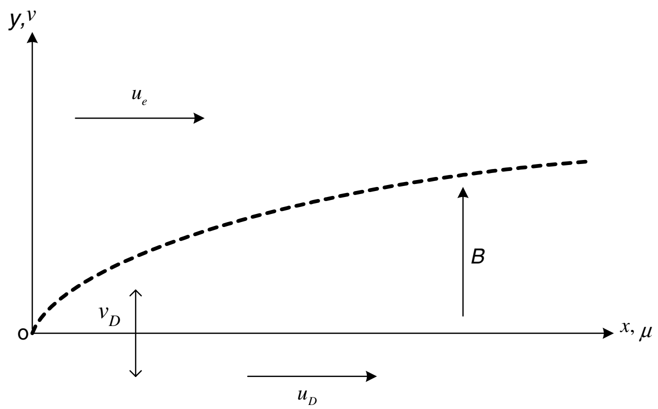

Fig. 1. Schematic diagram of the mathematical model of the problem under investigation and the co-ordinate system employed.

the surface. The parameter $M = \frac{\sigma B_0^2}{u_\infty \rho}$ is the magnetic parameter, where σ and ρ are the electric conductivity and the fluid density, respectively. Eq. (1) will be solved subject to the boundary conditions:

$$f(0) = a, \quad f'(0) = b \tag{2}$$

and

$$f'(\infty) = \lim_{\eta \to \infty} f'(\eta) = 1. \tag{3}$$

The parameters a and b are given by $a = \frac{2v_w}{(m+1)\sqrt{\nu u_\infty}}$ and $b = \frac{u_w}{u_\infty}$, where ν is the kinematic viscosity of the fluid and v_w is the suction/injection parameter. Here, it is assumed that the suction/injection velocity is given by $v_s(x) = -v_w x^{\frac{m-1}{2}}$. The case $v_w < 0$ corresponds to suction and $v_w > 0$ to injection of the fluid. The case $v_w = 0$ characterizes an impermeable surface. The derivation of (1)–(3) as well as a complete physical interpretation of the problem can be found in [14,41,43,45]. In Chiam [18] reported that similarity solutions are possible if the magnetic field has the special form ($B(x) = B_0 x^{\frac{m-1}{2}}$). Very recently Anjali and Thiyagarajan [4] considered the nonlinear MHD flow and heat transfer in which the magnetic field satisfies also this special form. In both papers, the free stream function is neglected ($u_\infty = 0$).

Numerical and analytical solutions of the MHD problem, in the absence of the free stream function ($f'(\infty) = 0$) were obtained in [15,20,40,44]. Numerical solutions, in the presence of the free stream velocity can be found in [3,41,45] for both momentum and heat transfers.

In a physical different but mathematically identical context, Eq. (1), with $M = -m$, which reads (by a scaling)

$$f''' + (1+m)ff'' + 2mf'(1-f') = 0 \tag{4}$$

has been investigated by Aly et al. [2], Brighi et al. [11], Brighi and Hoernel [12], Guedda [29], Magyari and Aly ([34,35]) and Nazar et al. [38]. This equation with the boundary condition ($a = 0, b = 1 + \varepsilon$)

$$f(0) = 0, \quad f'(0) = 1 + \varepsilon, \quad f'(\infty) = 1 \tag{5}$$

arises in the modelling of the mixed convection boundary-layer flow in a porous medium. In [2] it is found that if m is positive and ε takes place in the rang $[\varepsilon_0, \infty)$, for some negative ε_0, there are two numerical solutions. The case $-1 \leqslant m \leqslant 0$ is also considered in [2]. The authors studied the problem for $\varepsilon_c \leqslant \varepsilon \leqslant 0.5$, for some $\varepsilon_c < 0$. It is shown that there exists ε_t such that the problem has two numerical solutions for $\varepsilon_c \leqslant \varepsilon \leqslant \varepsilon_t$. In Guedda [29], has investigated the theoretical analysis of (4) and (5). It was shown that, if $-1 < m < 0$ and $-1 < \varepsilon < 1/2$, there is an infinite number of solutions, which indeed motivated the present work. Some new interesting results on the uniqueness of concave and convex solutions to (4) and (5), for $m > 0$ and $\varepsilon > -1$, were reported in [12].

The purpose of the this note is to re-examine problem (1)–(3) for $-1 < m < -M < 0$. For specific values of b it is obtained a family of solutions parameterized by $\gamma = f''(0)$, for any $a \geqslant 0$. The nonexistence result is also given for $m \leqslant -1$.