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Forced Korteweg–de Vries equation in an elastic tube filled with an inviscid fluid

Tay Kim Gaik *

Panel of Mathematics, Sciences Studies Center, University College Technology of Tun Hussein Onn, 86400 Parit Raja, Batu Pahat, Johor, Malaysia

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Abstract

In the present work, treating the arteries as a prestressed thin walled elastic tube with a stenosis and the blood as an inviscid fluid, we have studied the propagation of weakly nonlinear waves in such a composite medium, in the long wave approximation, by use of the reductive perturbation method [C.S. Gardner, G.K. Morikawa, Similarity in the asymptotic behavior of collision-free hydromagnetic waves and water waves, Courant Institute Math. Sci. Report, NYO-9082 (1960) 1–30, T. Taniuti, C.C. Wei, Reductive perturbation method in non-linear wave propagation I, J. Phys. Soc. Jpn., 24 (1968) 941–946]. We obtained the forced Korteweg–de Vries (FKdV) equation with variable coefficients as the evolution equation. By use of the coordinate transformation, it is shown that this type of evolution equation admits a progressive wave solution with variable wave speed. As might be expected from physical consideration, the wave speed reaches its maximum value at the center of stenosis and gets smaller and smaller as we go away from the center of the stenosis. The variations of radial displacement and the fluid pressure with the distance parameter are also examined numerically. The results seem to be consistent with Bernoulli's law for inviscid fluid.
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1. Introduction

Due to its applications in arterial mechanics, the propagation of pressure pulses in fluid-filled distensible tubes has been studied by several researchers [1,2]. Most of the works on wave propagation in compliant tubes have considered small amplitude waves ignoring the nonlinear effects and focused on the dispersive character of waves (see [3–5]). However, when the nonlinear terms arising from the constitutive equations and kinematical relations are introduced, one has to consider either finite amplitude, or small-but-finite amplitude waves, depending on the order of nonlinearity.

* Tel.: +07 4536721/4536701; fax: +07 4536051.
E-mail address: tay@kuittho.edu.my

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The propagation of finite amplitude waves in fluid-filled elastic or viscoelastic tubes has been examined, for instance, by Rudinger [6], Ling and Atabek [7], Anliker et al. [8] and Tait and Moodie [9] by using the method of characteristics, in studying the shock formation. On the other hand, the propagation of small-but-finite amplitude waves in distensible tubes has been investigated by Johnson [10], Hashizume [11,12], and Yomosa [13]. In all these works [10–13], the effect of initial deformation is neglected. Recently in a series of works of Demiray, Antar and Bakirtas (see [14–28]) in which they treated artery as incompressible prestressed thin isotropic elastic, thick viscoelastic or tapered elastic tube filled with inviscid, viscous or layered fluid as blood, using approximate method on fluid equations and reductive perturbation method in the long-wave approximation, they obtained various evolution equations of Korteweg–de Vries, Burgers and Korteweg–de Vries–Burgers type equations. In all previous works, they treated the arteries as circularly cylindrical long thin tubes with a constant cross-section. However due to decomposition of fat or cholesterol in artery over time, the artery become narrower and may have variable radius along the axis of the tube.

Thus, in this work, treating the arteries as an incompressible prestressed thin walled elastic tube with a stenosis and the blood as an incompressible inviscid fluid, we have studied the propagation of weakly nonlinear waves in such a composite medium, in the long wave approximation, by use of the reductive perturbation method [29,30]. We obtained the forced Korteweg–de Vries (FKdV) equation with variable coefficients as the evolution equation. By use of the coordinate transformation, it is shown that this type of evolution equation admits a progressive wave solution with variable wave speed. As might be expected from physical consideration, the wave speed reaches its maximum value at the center of stenosis and gets smaller and smaller as one goes away from the center of the stenosis. The variations of radial displacement and the fluid pressure with the distance parameter are also examined numerically. The results seem to be consistent with Bernoulli's law for inviscid fluid.

2. Basic equations and theoretical preliminaries

In this section, we shall give the derivation of the field equations of an elastic tube, which is considered to be a model for an artery, and an inviscid fluid, which is considered to be a model for blood.

2.1. Equations of tube

In this sub-section, we shall derive the governing equations of an elastic tube filled with an inviscid fluid. Such a combination of a solid and a fluid is considered to be a model for blood flow in arteries.

For a healthy human being, the systolic pressure is about 120 mm Hg, and the diastolic pressure is around 80 mm Hg. This means that the arteries are subjected to a mean pressure $P_0 = 100$ mm Hg, and in the course of blood flow, a dynamical pressure increment $\Delta P = \pm 20$ mm Hg is added on this initial field. Moreover, experimental studies [2] revealed that the arteries are also subjected to an initial axial stretch λ_z, which is about $\lambda_z = 1.6$. These observations show that the arteries are initially subjected to static deformation both in the radial and the axial directions, and a dynamical pressure (or a radial displacement u^*) is superimposed on this initial deformation. Due to the external tethering in the axial direction, the effect of axial displacement is neglected.

Now, we consider a thin and long tube of circular cross-section with radius $R^*(Z^*)$ in the cylindrical polar coordinates (R^*, Θ, Z^*). Then, the position vector of a point on the tube may be described by

$$\mathbf{R} = R^*(Z^*)\mathbf{e}_r + Z^*\mathbf{e}_z, \tag{1}$$

where $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_z$ are the unit base vectors in the cylindrical polar coordinates and Z^* is the axial coordinates of a material point in the natural state.

The arclengths along the meridional and circumferential curves are given by

$$\mathrm{d}S_Z = \left[1 + \left(\frac{\mathrm{d}R^*}{\mathrm{d}Z^*}\right)^2\right]^{1/2} \mathrm{d}Z^*, \quad \mathrm{d}S_\Theta = R^*(Z^*)\,\mathrm{d}\Theta. \tag{2}$$

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