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# Propagation of singular surfaces in thermo-microstretch continua with memory

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**Abstract**

This paper concerns the behaviour of singular surfaces which propagate in a thermo-microstretch viscoelastic body. First, the necessary and sufficient conditions that the infinitesimal entropy production be invariant under time-reversal are presented. Then, the propagation conditions and growth equations which govern the evolution of singular surfaces of order 1 are studied.


## 1. Introduction

In order to describe adequately the behaviour of materials such as liquid crystals, fluid suspensions, polycrystalline aggregates and granular media it is necessary to introduce into the continuum theory terms reflecting the microstructure of the materials (see e.g. [1,2]). In the framework of micromorphic theory introduced by Eringen [2,3], a material point is endowed with three deformable directors. When the directors are constrained to have only breathing-type microdeformations, then the body is a microstretch continuum [2,4]. The material points of the microstretch bodies can stretch and contract independently of their translations and rotations. The theory is expected to find applications in the treatment of the mechanics of composites materials reinforced with chopped fibers and various porous materials. The theory of microstretch continua is a generalization of the theory of Cosserat continua. The history of motion is important for rheological materials and in the dissipation and relaxation phenomena it plays a central role. In recent years new investigations in the theory of materials with memory have been made (see e.g. [2,5,6]). The theory of micromorphic materials with memory has been developed by Eringen [2,7]. The propagation conditions and growth equations which govern the propagation of waves in micropolar viscoelastic bodies have been derived and discussed by McCarthy and Eringen [8]. In the classical theory of viscoelastic materials, Day [9] proved that the work done in every closed strain path starting from zero is invariant under time-reversal if and only if the stress relaxation function is

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symmetric. In [10], Gurtin derived an extension of Day's result within the context of the thermodynamics of materials with memory.

The present article is concerned with a linear theory of thermoviscoelastic micromorphic continua. In the first part of the paper we use the results established by Day [9] and Gurtin [10] to present the necessary and sufficient conditions that the infinitesimal entropy production be invariant under time-reversal. We derive a linear theory of thermoviscoelastic micromorphic continua in which the heat flux is independent on the present temperature gradient, but depends upon the past theory of this gradient. The theory permits propagation of thermal waves at finite speed. Moreover, in this theory the conductivity tensor is symmetric. In the second part of the paper we study the propagation conditions and growth equations which govern the propagation of singular surfaces of order 1 in homogeneous and isotropic thermoviscoelastic microstretch materials.

In Section 2, we present the local form of balance laws in nonlinear theory of thermodynamics of micromorphic continua. Section 3 deals with the restrictions imposed on the constitutive equations of the linear theory by the invariance under time-reversal. In Section 4, we consider the linear theory of microstretch continua and investigate the singular surfaces of order 1. Section 5 is devoted to the study of the decay laws.

## 2. Balance laws

In this section, we present the local form of the balance laws in the framework of micromorphic theory of continua. We consider a body that at time $t^0$ occupies the region $B$ of euclidean three-dimensional space and is bounded by the piecewise smooth surface $\partial B$. The motion of the body is referred to the reference configuration $B$ and a fixed system of rectangular cartesian axes. We identify a typical particle of the body with its position $\boldsymbol{X}$ in the reference configuration. Letters in boldface stand for tensors of an order $p \geqslant 1$, and if $\boldsymbol{v}$ has the order $p$, we write $v_{ij\ldots k}$ ($p$ subscripts) for the components of $\boldsymbol{v}$ in the underlying rectangular cartesian coordinate. We shall employ the usual summation and differentiation conventions: Latin subscripts are understood to range over the integers $(1,2,3)$ whereas Greek subscripts are confined to the range $(1,2)$, summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to corresponding material coordinate. Throughout this paper, a superposed dot denotes the material derivative with respect to the time $t$. The position of a typical particle at time $t$ is $\boldsymbol{x}$. We consider a continuum with microstructure each material point of which has 12 degrees of freedom. The deformation is described by [2]

$$x_i = x_i(X_j, t), \quad \chi_{ik} = \chi_{ik}(X_j, t), \quad (X_j, t) \in B \times I, \tag{2.1}$$

where $\chi_{ik}$ are the microdeformation functions and $I$ is a time interval. The functions (2.1) are assumed to be sufficiently smooth for the ensuing analysis to be valid. Moreover,

$$\det(x_{i,j}) > 0, \quad \det \chi_{ij} > 0.$$

The equations of motion can be expressed as

$$T^{(1)}_{ji,j} + \rho_0 f_i = \rho_0 \ddot{x}_i, \quad M^{(1)}_{kij,k} - S^{(1)}_{ij} + \rho_0 L_{ij} = I_{jm} \ddot{\chi}_{im}, \tag{2.2}$$

on $B \times I$, where $T^{(1)}_{ji}$ is the first Piola–Kirchhoff stress tensor, $M^{(1)}_{kij}$ is the stress moment tensor of Piola–Kirchhoff type, $S^{(1)}_{ij}$ is a generalized stress, $\rho_0$ is the mass density at time $t^0$, $I_{jk}$ are coefficients of inertia, and $f_i$ and $L_{ij}$ are body loads.

Let $e$ denote the internal energy of the body per unit mass, and let $s$ denote the external heat supply per unit mass, per unit time. The local form of the law of balance of energy is

$$\rho_0 \dot{e} = T^{(1)}_{ji} \dot{x}_{i,j} + S^{(1)}_{ij} \dot{\chi}_{ij} + M^{(1)}_{kij} \dot{\chi}_{ij,k} + \rho_0 s + Q_{i,i}, \tag{2.3}$$

where $Q$ is the heat flux vector. We denote by $\eta$ the entropy per unit mass and by $T$ the absolute temperature. The entropy production rate is defined by

$$\gamma = \rho_0 \dot{\eta} - \frac{1}{T} \rho_0 s - \left( \frac{1}{T} Q_i \right)_{,i}. \tag{2.4}$$