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The effect of transpiration on self-similar boundary layer flow over moving surfaces

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Abstract

The simultaneous effects of normal transpiration through and tangential movement of a semi-infinite plate on self-similar boundary layer flow beneath a uniform free stream is considered. The flow is therefore governed by a plate velocity parameter λ and a transpiration parameter μ and the computed wall shear stress parameter is $f''(0)$. Dual solutions are found for each value of μ in $\lambda - f''(0)$ parameter space. It is shown that the range of known dual solutions for zero transpiration increases with suction and decreases with blowing. A stability analysis for this self-similar flow reveals that, for each value of μ, lower solution branches are unstable while upper solution branches are stable.

Keywords: Similarity solutions; Transpiration; Nonuniqueness; Stability

1. Introduction

Over the past several decades a number of studies have shown the existence of multiple solutions of boundary layer flows driven by moving surfaces with and without external pressure gradients. In this paper we consider uniform flow over a belt that moves into or out of the origin at uniform speed. This relates to the pioneering works by Klemp and Acrivos [1,2] who considered the motion induced by finite and semi-infinite flat plates moving at constant velocity beneath a uniform mainstream. In the latter case for which a similarity reduction to an ordinary differential equation is available, dual solutions were found when the plate advances toward the oncoming stream. Hussaini et al. [3] proved nonuniqueness of the similarity solutions for this case. As part of a more general study on Falkner–Skan flows with stretching boundaries, Riley and Weidman [4] analyzed the nature of the lower branch dual solution as the shear stress and plate velocity simultaneously tend to zero, showing that the boundary layer lifts off the plate, even in the absence of blowing.

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A problem mathematically equivalent to viscous uniform flow over a moving plate occurs in mixed convection boundary layer flow in a fluid-saturated porous medium bounded by a rigid semi-infinite heated vertical plate. The similarity equations involve a nondimensional parameter which describes the relative importance of natural to forced convection of which there are two basic configurations. Firstly, when buoyancy acts in the same direction as the uniform external flow, the solutions are unique; see Cheng [5]. Secondly, when buoyancy opposes the uniform external flow, Merkin [6] found the same dual solutions cited in [1–4] above. Pertinent to the present investigation is the follow-on study by Merkin [7] who analyzed the stability of the dual solutions, showing that the lower branch is unstable whilst the upper branch is stable.

The focus of the present endeavor is twofold. First we consider the simultaneous effects of transpiration, characterized by parameter μ, and plate movement, characterized by parameter λ, on the self-similar zero pressure gradient boundary layer flow described above. Second, the stability of the dual solutions based on linear disturbances of the steady similarity solutions is determined.

The presentation proceeds as follows. In Section 2 the boundary layer problem for flow past a semi-infinite porous plate moving toward or away from the origin beneath a uniform stream is formulated and solved numerically. Asymptotic and perturbation solutions are provided to elucidate the nature of solutions near critical points in Sections 2.1–2.3. In Section 2.4 the stability of dual solutions is determined following the method of Merkin [7]. A discussion of results and concluding remarks are given in Section 3.

2. Formulation and solution

When an external stream of uniform speed U flows parallel to a semi-infinite plate located at $y = 0$ for all $x \geqslant 0$, the flow is one of zero pressure gradient. The streamwise and plate-normal velocities are u and v, respectively. The dimensional unsteady Prandtl boundary layer equations governing this flow are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}. \tag{1b}$$

We first consider steady flow solutions of (1). The well-known Blasius similarity transformation [8]

$$u(x,y) = Uf'(\eta), \qquad v(x,y) = \sqrt{\frac{\nu U}{2x}}(\eta f' - f), \qquad \eta = y\sqrt{\frac{U}{2\nu x}} \tag{2}$$

is valid for plates receding into or moving out of the origin at uniform speed λU. Transpiration through porous plates is also admitted. The normal velocity in (2) shows that this fluid injection/suction velocity must be of the form

$$v_0(x) = -\sqrt{\frac{\nu U}{2x}}f(0). \tag{3}$$

Substituting *ansatz* (2) into (1) yields the boundary-value problem

$$f''' + ff'' = 0. \tag{4a}$$

$$f(0) = \mu, \qquad f'(0) = \lambda, \qquad f'(\infty) = 1, \tag{4b, c, d}$$

where $\mu > 0$ corresponds to suction and $\lambda > 0$ corresponds to downstream movement of the plate from the origin. Solutions for $f''(0)$, at each value of μ and λ, are related to the wall shear stress $\mathscr{T}$ through the expression

$$\mathscr{T} = \rho\nu\frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{(\rho U)^{3/2}\nu^{1/2}}{\sqrt{2}}\frac{f''(0)}{x^{1/2}}$$

showing that the shear stress decreases as the inverse square root of the downstream coordinate.

We denote critical points in $\lambda - f''(0)$ space using $\{\lambda, f''(0)\}$. The obvious exact solution $f(\eta) = \mu + \eta$, valid for all transpirations μ, exists at the $\{1, 0\}$ focal point of the system. Solutions for $\mu = 0$ are those analyzed