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Steady-state vibrations for the state of generalized plane strain in a linear piezoelectric medium

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Abstract

In this paper, steady-state vibrations are studied for the state of generalized plane strain in a linear piezoelectric medium. The fundamental boundary value problems are stated for vibrations in a prismatic piezoelectric body with arbitrary cross-section composed of material with the more general, tetragonal $\bar{4}$, symmetry . Along with general theory, we present some other useful results concerning the behavior of the matrix of fundamental solutions. The conditions for the critical values of the field quantities on the boundary are derived.

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Keywords: Piezoelectricity; Generalized plane strain; Steady-state; Resonance; Existence; Fundamental solutions

1. Introduction

Modern developments of micro-electrical–mechanical systems, miniaturized power sources and other devices, for example, piezomotors have renewed interest in the fundamental theory of linear piezoelectric materials and their applications [6,7,11–13,15,16]. The first publications on the applications of piezoelectricity and the development of the theory of vibrations in piezoelectric solids began to appear in the early part of the twentieth century [1,2]. The use of piezoelectric materials in micro power systems produces significant advantages due to their light weight, superior energy conversion efficiency and energy density [19]. Since a variety of piezoelectric devices operates on resonant frequencies such as piezoelectric transformers, actuators, resonators etc [3,5,20], the investigation of the nature of steady-state vibrations is of great importance. In [23] some problems of vibrations of piezoelectric plates were investigated. To this end, in this paper, we use boundary integral

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equation methods to establish the solvability of boundary value problems for the steady-state vibrations of a prismatic piezoelectric body of smooth yet arbitrary cross-section. As a complementary result of this method the conditions for the critical values of the field quantities on the boundary are obtained.

2. Basic equations

In what follows Greek and Latin indices take the values 1, 2, and 1, 2, 3, respectively, the convention of summation over repeated indices is understood, $\mathscr{M}_{m\times n}$ is the space of $(m\times n)$-matrices, a superscript T indicates matrix transposition and $(\ldots)_{,\alpha}\equiv\partial(\ldots)/\partial x_\alpha$. For $A\in\mathscr{M}_{m\times n}$ we denote the mth row of the element A by $A_{(m)}$ and the nth column as $A^{(n)}$. Also, if X is a space of scalar functions and v is a matrix, $v\in X$ means that every component of v belongs to X.

Let Ω be an infinite cylinder $\Omega=\{\mathbf{x}\in\mathbb{R}^3:(x_1,x_2)\in S^+\}$ where S^+ is a simply-connected domain of $\mathbb{R}^2$ such that its boundary ∂S is sufficiently smooth. Let Ω be occupied by a homogeneous anisotropic linearly piezoelectric material. The equations of motion and charge equation in the case of plane piezoelectricity (generalized plane strain) are given by [8]:

$$\begin{aligned}&C_{i\alpha k\beta}u_{k,\alpha\beta}+e_{vi\alpha}\phi_{,v\alpha}=\rho\ddot{u}_i-f_i,\\&-e_{\gamma i\alpha}u_{i,\gamma\alpha}+\epsilon_{\alpha\beta}\phi_{,\alpha\beta}=-q,\end{aligned}\tag{2.1}$$

where u_i are the components of the mechanical displacement vector field, f_i are components of external force, q is the external charge, ϕ is the electric potential such that the electric field $\mathbf{E}$ is given by $\mathbf{E}=-\nabla\phi$, and $C_{i\alpha k\beta}$, $e_{vi\alpha}$, $\epsilon_{\alpha\beta}$ are, respectively, the elastic, piezoelectric and electric permittivity constants of the material. Without loss of generality we will assume that f_i and q are zero since we can always construct the solution of (2.1) as a sum of the solution of homogeneous system and a particular solution of the form [10,21,22]:

$$\tilde{u}(x)=\frac{1}{2}\int_{S^+}\Gamma(x,y,\omega)F(y)\,\mathrm{d}y,$$

where $\Gamma(x,y,\omega)$ is the matrix of fundamental solutions corresponding to the homogeneous system (2.1) and $F(x)=(f_1,f_2,f_3,-q)$. In the case of steady-state vibrations we expect the displacement components and electric potential to be of the form:

$$\begin{aligned}&u_i(x,t)=\mathrm{Re}(u_i(x)\mathrm{e}^{\mathrm{i}\omega t}),\quad u_i(x)=u_i^{(1)}(x)+\mathrm{i}u_i^{(2)}(x),\\&\phi(x,t)=\mathrm{Re}(\phi(x)\mathrm{e}^{\mathrm{i}\omega t}),\quad \phi(x)=\phi^{(1)}(x)+\mathrm{i}\phi^{(2)}(x),\quad x=(x_1,x_2).\end{aligned}\tag{2.2}$$

Substitution of (2.2) in (2.1) gives:

$$\begin{aligned}&C_{i\alpha k\beta}u_{k,\alpha\beta}+e_{vi\alpha}\phi_{,v\alpha}+\rho\omega^2u_i=0,\\&-e_{\gamma i\alpha}u_{i,\gamma\alpha}+\epsilon_{\alpha\beta}\phi_{,\alpha\beta}=0.\end{aligned}\tag{2.3}$$

Eq. (2.3) represent the governing system of equations describing steady-state vibrations in the context of (generalized) plane piezoelectricity. The appropriate boundary conditions for (2.3) are given by [8]:

$$\begin{cases}(C_{i\alpha k\beta}u_{k,\beta}+e_{vi\alpha}\phi_{,v})n_\alpha=t_i^*(x),\\-(e_{\gamma i\alpha}u_{,\gamma}-\epsilon_{\alpha\beta}\phi_{,\beta})n_\alpha=D^*(x)\quad\text{on }\partial S\end{cases}\tag{2.4}$$

in the case of the Neumann problem, and

$$\begin{cases}u_i(x)=u_i^*(x),\\\phi(x)=\phi^*(x)\quad\text{on }\partial S\end{cases}\tag{2.5}$$

in the case of the Dirichlet problem. Here D^*, t_i^*, u_i^*, ϕ^* are prescribed functions on ∂S and n_α are components of the outward normal n to ∂S. In the particular case of tetragonal $\bar{4}$ material [18] the system (2.3) becomes