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SCIENCE DIRECT
International Journal of Engineering Science 44 (2006) 574–594
International Journal of Engineering Science
www.elsevier.com/locate/ijengsci

A finite-strain elastic–plastic Cosserat theory for polycrystals with grain rotations

Patrizio Neff *

AG6, Fachbereich Mathematik, Darmstadt University of Technology, Schlossgartenstrasse 7, 64289 Darmstadt, Germany

Received 6 September 2005; received in revised form 27 February 2006; accepted 1 April 2006
Available online 12 July 2006

(Communicated by R.W. OGDEN)

Abstract

We investigate geometrically exact generalized continua of Cosserat micropolar type. A variational form of these models is recalled and extended to finite-strain elasto-plasticity based on the multiplicative decomposition of the deformation gradient. The stress driving the plastic evolution is the Eshelby energy momentum tensor. No plastic Cosserat rotation is introduced and the plastic spin is set to zero. It is argued that the traditional Cosserat couple modulus μ_c should be set to zero for polycrystal specimens liable to fracture in shear, still leading to a complete Cosserat theory with independent rotations in the geometrically exact case in contrast to the infinitesimal, linearized model. A geometrical linearization of the presented finite-strain plasticity model is already shown to be well posed.

MSC: 74A35; 74C05; 74C20; 74E15

Keywords: Plasticity; Viscoplasticity; Polar-materials; Microstructure

1. Introduction

1.1. The development of Cosserat models, motivation and applications

This article addresses the extension of a **geometrically exact** generalized continua of **Cosserat micropolar** type to elasto-plasticity. General continuum models involving **independent rotations** have been introduced by the Cosserat brothers [18]. Their development has been rediscovered in the beginning of the 1960s

* Tel.: +49 6151 16 3495; fax: +49 6151 16 4011.
E-mail address: neff@mathematik.tu-darmstadt.de

0020-7225/$ - see front matter
doi:10.1016/j.ijengsci.2006.04.002

[99,50,1,36,33,116,117,48,77,107,118]. At that time theoretical investigations on non-classical continuum theories were the main motivation [59]. The Cosserat concept has been generalized in various directions, for an overview of these so called **microcontinuum** theories we refer to [35,32,101,14,13]. In a series of papers [90,87,84,83,88,93,89,94] the author has investigated the mathematical and physical aspects of generalized microcontinuum theories in the elastic case.

Among the first contributions extending the Cosserat framework to (infinitesimal, geometrically linear) elasto-plasticity we mention [105,67,10]. More recent (infinitesimal) elastic–plastic formulations have been investigated in [22,27,53,100]. These models directly comprise of elastic **and** plastic Cosserat effects. More recently, the models have been extended to a finite-strain elastic–plastic setting, see e.g. [46,102–104,113,45,40]. Most of these finite-strain formulations comprise of elastic and plastic Cosserat effects as well together with an additional split of the curvature into elastic and plastic parts. In these models, the permanent, locally averaged plastic grain rotation (‘‘plastic spin’’) is identified with a plastic Cosserat rotation [68]. However, the physical and mathematical significance of such a plastic Cosserat rotation seems to be more difficult to assess[1] than models in which Cosserat effects are restricted to the elastic response of the material [40]. This contribution is of the last type.

Our working hypothesis is that couple stresses, understood as the presence of non-symmetric parts of the Cauchy stress, exist in real elastic material [118]. Discrepancies between classical linearized elasticity theory, not accounting for couple stresses, and experiments are observed predominantly for high gradients which occur by stress concentrations in the neighbourhood of holes, notches and cracks as predicted by classical elasticity. Indeed, the measured stresses around cracks are smaller than the predicted ones. However, the source, magnitude and significance of couple stresses are still discussed. A group of researchers [119,52,57], supported by experimental evidence [44,43,108,31] admits **elastic** couple stresses in elastostatics on a **macroscale** only due to (very small) non-local effects such that an infinitesimal-strain elastic Cosserat micropolar theory is meaningless: the infinitesimal continuum rotations must coincide with the infinitesimal microrotations and moreover, couple stresses are altogether neglected since they are supposedly small.[2] Another group of researchers uses the infinitesimal Cosserat micropolar model, admitting non-symmetric infinitesimal constitutive Cauchy stresses as a first order effect due to independent infinitesimal microrotations [113,53,79,21,46,47]. Infinitesimal elasto-plastic extensions of the indeterminate couple-stress theory (constraint Cosserat model) have been investigated in [38,100]. Apparently, both views exclude each other.[3]

Notwithstanding, we present a model perhaps reconciling both views: the controversy may be due to the uncritical use of the infinitesimal, geometrically linear Cosserat model but disappears for a geometrically exact description of the Cosserat theory. The **Cosserat couple modulus** μ_c (modulus of local rotational stiffness, Cosserat shear modulus, torsional rigidity cf. (2.5)) possibly appearing in both the infinitesimal and geometrically

[1] ‘‘Freilich hat sich in den letzten Jahren mein Pessimismus hinsichtlich einer praktischen Anwendung der Cosserat–Theorien und ihrer Ableger verstärkt. Alle Ehrfahrung in der Bodenmechanik und bei der plastischen Umformung von Metallen, wo man die Cosserat Drehung (‘plastischer Spin’) mit der bleibenden, räumlich gemittelten Drehung von Körnern identifizieren kann, deuten darauf hin, dass der beobachtbare Effekt größenordnungsmäßig innerhalb des experimentellen Streubereiches liegt – und das läßt sich auch plausibel erklären, kann also kaum verbessert werden. Freilich stellt dies keine besondere Ermutigung für weitere Untersuchungen dar. Übrigens gilt dies für alle Theorien, die die mittlere lokale Gefügedrehung in eine makroskopische Theorie einbauen, und wahrscheinlich auch für andere mikropolare Ansätze’’. My Translation: ‘‘However in the last years I have become more pessimistic as regards the practical applications of Cosserat models and its variants. All experience in soil mechanics and the plastic deformations of metals, where Cosserat rotations are identified with the remaining, spatially averaged rotations of grains (‘plastic spin’), indicates, that the observed effect is of the order of the experimental accuracy and this cannot be improved. This does not represent a stimulus for further investigations. Moreover, this holds for all theories which account for local rotations in an averaged sense and probably for other micropolar variants as well’’. personal communication with Lippmann [69].

[2] In [52, p. 339] we read: ‘‘Momentenspannungen sind merklich erst in Bereichen vorhanden, in denen normalerweise nicht nur die Anwendung der linearen, sondern auch der nichtlinearen Elastizitätstheorie nicht mehr sinnvoll ist’’. My translation: ‘‘Couple stresses are noticeable only in regions, in which usually not only the application of the linear, but also of the nonlinear elasticity model is in doubt’’.

[3] The experimental results reported in [38] on the torsion of thin copper wires revealed a strong **geometrical length scale effect** of the **plastic** behaviour: the thinnest wires displayed comparatively the strongest response up into the plastic range. Whether this is due to a genuine Cosserat effect cannot be ascertained; in their experiments, also **grain size effects** interfered: the different wires had different grain sizes.