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On the steady shear flow of a dipolar fluid in a porous half-space

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Abstract

The plate-driven flow of an incompressible, homogeneous, dipolar fluid in a porous half-space is studied. The exact solution to this problem, which corresponds to the porous media version of the steady-state Stokes' first problem for dipolar fluids, is obtained and an in-depth analytical and numerical investigation is carried out. Specifically, special cases of the physical parameters are determined and investigated, asymptotic results are given, physical interpretation(s) are discussed, and connections to other areas are noted.

Most significantly, the analysis shows the following: (I) that a back flow can occur; (II) the flow velocity is predicted to exceed that of the (driving) plate for certain values of the physical parameters; (III) that a connection exists between the golden ratio and the solution presented here; (IV) that the flow is mathematically equivalent to the unforced motion of a damped harmonic oscillator.

Keywords: Dipolar fluids; Darcy's law; Stokes' first problem; Back flow; Golden ratio

1. Introduction

In 1967, Bleustein and Green [1] presented the theory of dipolar fluids, the simplest example of a class of non-Newtonian fluids known as multipolar fluids. Cowin [2] has pointed out that dipolar fluids are a special case of fluids with deformable microstructure. He also noted similarities between the equations governing dipolar fluids and those of polar fluids in certain flow configurations. Straughan [3] has suggested that the theory of dipolar fluids should be capable of describing fluids made up of long molecules or possibly a suspension of long molecular particles. In dipolar fluid theory, the effects of this microstructure are manifested through the presence of the material constants d and l, known as the dipolar constants, and the dipolar body force in the equations of motion.

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In 1970, Green and Naghdi [4] proposed an alternative form of dipolar inertia. Guram [5] considered Stokes' first problem for the special case $d = l$. Straughan [3] studied the nonlinear stability problem for the case in which a layer of dipolar fluid is heated from below. The present authors have studied Stokes' first problem for a dipolar fluid, with and without heat conduction, for arbitrary values of the dipolar constants [6,7]. In 2002, these authors [8] derived exact solutions to the unsteady plane Couette flow problem for dipolar fluids and provided a detailed listing of works on dipolar fluids. Later, in 2003, they also investigated Stokes' second problem for dipolar fluids under two different boundary conditions and also gave the solution to this problem for both couple stresses and second grade fluids [9]. More recently, Puri [10] carried out a linear stability analysis of dipolar flow between two parallel plates and Quintanilla and Straughan [11] compared/contrasted the theory of dipolar fluids with those of Green and Naghdi and Camassa–Holm fluids in the context of a nonlinear stability study.

Stokes' first problem, also known as Rayleigh's problem, was first solved by Stokes in 1851. This unsteady flow problem examines the diffusion of vorticity in a half-space filled with a viscous, incompressible fluid that is set in motion when an infinite flat plate (i.e., the bounding plane) suddenly assumes a constant velocity parallel to itself from rest. While Stokes' investigation naturally pertained to Newtonian fluids, many researchers have, over the years, studied a variety of non-Newtonian fluids in the context of this problem and several variations of it. The main reason why this problem has attracted so much attention is due to the geometry involved, which results in a one-dimensional equation of motion that is linear, but nevertheless exact, if the fluid's constitutive relation is linear (see, e.g., [5–7,12–17] and the references therein).

In this work, we have replaced the open half-space of the original Stokes' first problem with one that is occupied by a porous material of constant permeability and porosity K (>0) and $\beta \in (0,1)$, respectively. To incorporate the effects of the pores on the velocity field, we have made use of Darcy's law [18]

$$\nabla P = -\frac{\mu\beta}{K}\mathbf{v}, \tag{1.1}$$

where P is an intrinsic pressure, the constant μ is the dynamic (or shear) viscosity, and the Darcy velocity vector $\mathscr{V}$ is related to the usual (i.e., volume-averaged over a volume element consisting of fluid only) velocity vector $\mathbf{v}$ by the Dupuit–Forchheimer relationship $\mathscr{V} = \beta\mathbf{v}$ [18]. Since the 1960s, interest in non-Newtonian flows through porous media has steadily grown due, in part, to the demands of such diverse fields as biorheology, geophysics, and the chemical and petroleum industries (see, e.g., [14,16–24] and the references therein).

Lastly, our objective in writing this article has been to present, for the first time that we are aware of, an in-depth analysis of steady-state dipolar flow in porous media. To this end, the paper is arranged as follows: In Section 2, the governing constitutive equations and balance laws are stated. In Section 3, the physical problem is stated, its mathematical formulation given, and the exact solution presented. In Sections 4 and 5, analytical and numerical results are presented, respectively. And finally, in Section 6, conclusions are stated, findings are discussed, and other applications of this research are noted.

2. Constitutive equations and balance laws

As given by Bleustein and Green [1], the (linearized) constitutive equations for an isothermal, isotropic, homogeneous incompressible dipolar fluid are

$$\tau_{ij} + \Phi\delta_{ij} = 2\mu d_{ij}, \tag{2.1}$$

$$\Sigma_{(ij)k} + \Psi_i\delta_{jk} + \Psi_j\delta_{ik} = h_1\delta_{ij}A_{kmm} + h_2(A_{ijk} + A_{jik}) + h_3A_{kji}, \tag{2.2}$$

where τ_{ij} is the stress tensor, $\Sigma_{(ij)k}$ are the components of the dipolar stress tensor (i.e., Σ_{ijk}) which are symmetric in the first two indices, the arbitrary functions Φ and Ψ_i govern the pressure and arise from the solenoidal nature of the velocity field, δ_{ij} is the Kronecker delta,

$$d_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) = d_{ji} \tag{2.3}$$

and

$$A_{ijk} = v_{i,jk} = A_{ikj}. \tag{2.4}$$