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Dielectric breakdown model for a conductive crack and electrode in piezoelectric materials

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Abstract

The strip dielectric breakdown (DB) model introduced by Zhang and Gao [T.Y. Zhang, C.F. Gao, Fracture behavior of piezoelectric materials, Thero. Appl. Fract. Mech. 41 (2004) 339–379] is used to study the generalized 2D problem of a conductive crack and an electrode in an infinite piezoelectric material. The energy release rate and stress intensity factors are derived based on the Stroh formalism, and then they are applied as failure criteria to predict the critical fracture loads. It is found that the DB strip may take the shielding effect on a conductive crack or electrode. For the case of an electrode, the local energy release rate and stress intensity factor become zero when DB happens ahead of the electrode tip. For the case of a mode-I conductive crack in a transversely isotropic piezoelectric solid, the results based on the DB model show that the critical stress intensity factor linearly increases as the applied electric field parallel to the poling direction increases, while it linearly decreases as the applied electric field anti-parallel to the poling direction increases. Finally, the upper and lower bounds of the actual critical fracture loads are proposed for a conductive crack in a piezoelectric material under combined mechanical–electrical loads.

Keywords: Piezoelectric materials; Crack; Electrode; Dielectric breakdown; Fracture and failure; Stroh formalism

1. Introduction

Recently, Zhang and Gao [1] presented a strip dielectric breakdown (DB) model based on the concept of dielectric breakdown to explore the effects of electrical non-linearity on piezoelectric fracture. The concept of dielectric breakdown was originally introduced to predict the failure of dielectrics in high voltage engineering and it has been widely used for the cases of dielectric materials [2–7]. For the case of piezoelectric materials, Zhang and Gao [1] developed the strip DB model in which the electric field is assumed to be a constant in a

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strip ahead of the crack tip, and then they used the DB model to study the generalized 2D problem of an impermeable crack in a piezoelectric material with the general linear constitutive equations [8,9].

In the present work we extend the application of the DB model to the generalized two-dimensional problem of a conductive crack and electrode in piezoelectric materials. Using the concept of dielectric breakdown, McMeeking [10] solved the problem of an electric field around a conductive crack in dielectrics, and Suo [11] addressed the failure behavior of conductive tubular channels in dielectric and ferroelectric ceramics. Ru [12] derived the stress intensity factors and the energy release rate of a conductive crack based on the strip polarization saturation model [13]. More recently, Wang and Zhang [14] studied the conductive crack problem in piezoelectric materials based upon electric field saturation concept. They gave the solution of energy release rate and stress intensity factors using the Fourier integral technology for a 2D problem. In the present work, we study the generalized 2D problems of a conductive crack and a finite electrode in piezoelectric materials using the DB model with the Stroh formalism. The solution for the electrode is first presented based on the DB model, while the problem of the conductive crack is further studied under rigorous mathematical manipulation with new insights into the field.

Below is the plane of this work. The Stroh formulism is outlined in Section 2, and then it is used in Section 3 to derive the local energy release and stress intensity factor for a conductive crack based on the DB model. These results are extended to the case of an electrode in a piezoelectric material in the Appendix. Several important results are given in Section 4, respectively, for a mode-I conductive crack and a mode-III conductive crack in a transversely isotropic piezoelectric ceramic. Discussions on the upper and lower bounds of critical fracture loads are then made for a conductive crack under combined mechanical–electrical loads. Finally, the presented work is concluded in Section 5.

2. The Stroh formalism

The Stroh formalism has been widely used in 2D anisotropic elasticity. The material in this section is not new, and it can be found in previous publications [15–19,1]. However, for self-containing purpose of this work and convenience for readers, below we still list the main equations to be used in the later analysis.

Consider a piezoelectric solid in a rectangular coordinate system, x_i $(i = 1, 2, 3)$. The complete set of basic equations for the linear solid can be expressed as

$$\sigma_{ij,j} = 0, \qquad D_{i,i} = 0, \tag{1}$$

$$\gamma_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad E_i = -\varphi_{,i}, \tag{2}$$

$$\sigma_{ij} = C_{ijkl}\gamma_{kl} - e_{kij}E_k, \qquad D_k = e_{kij}\gamma_{ij} + \varepsilon_{kl}E_l, \tag{3}$$

where u_i, φ, σ_{ij}, γ_{ij}, D_j and E_i are the displacement, the electric potential, the stress, the strain, the electric displacement and the electric field, respectively, and C_{ijkl}, e_{ijk} and ε_{ij} stand for the elastic constants, the piezoelectric constants and the dielectric constants, respectively.

For generalized two-dimensional deformations in which u_i $(i = 1, 2, 3)$ and φ depend on x_1 and x_2 only, the general solution (the standard Stroh formalism) to Eqs. (1)–(3) can be expressed as

$$\mathbf{u} = \mathbf{A}\mathbf{f}(z) + \bar{\mathbf{A}}\overline{\mathbf{f}(z)}, \qquad \boldsymbol{\phi} = \mathbf{B}\mathbf{f}(z) + \overline{\mathbf{B}}\overline{\mathbf{f}(z)}, \tag{4}$$

where $\mathbf{u} = [u_1, u_2, u_3, \varphi]^{\mathrm{T}}$ is a generalized displacement vector, $\boldsymbol{\phi} = [\phi_1, \phi_2, \phi_3, \phi_4]^{\mathrm{T}}$ is a generalized stress vector, $\mathbf{A}$ and $\mathbf{B}$ are constant matrices, and $\mathbf{f}(z)$ is an unknown complex vector.

For impermeable cracks [20] or permeable cracks [21,22], the standard Stroh formalism above is convenient since the displacement vector, $\mathbf{u}$, is expressed in terms of the displacements and electric potential. For a conductive crack, we redefine a hybrid displacement vector, $\hat{\mathbf{u}}$, and a hybrid stress function vector, $\hat{\boldsymbol{\phi}}$, as

$$\hat{\mathbf{u}} = [u_1, u_2, u_3, \phi_4]^{\mathrm{T}}, \qquad \hat{\boldsymbol{\phi}} = [\phi_1, \phi_2, \phi_3, \varphi]^{\mathrm{T}}. \tag{5}$$

The hybrid displacement and stress function vectors can be expressed in terms of the original displacement and stress function vectors:

$$\hat{\mathbf{u}} = \mathbf{I}_u\mathbf{u} + \mathbf{I}_t\boldsymbol{\phi}, \qquad \hat{\boldsymbol{\phi}} = \mathbf{I}_t\mathbf{u} + \mathbf{I}_u\boldsymbol{\phi}, \tag{6}$$