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Magnetohydrodynamic (MHD) flows of viscoelastic fluids in converging/diverging channels

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Abstract

The present work is a theoretical investigation of the applicability of magnetic fields for controlling hydrodynamic separation in Jeffrey-Hamel flows of viscoelastic fluids. To achieve this goal, a local similarity solution was found for laminar, two-dimensional flow of a viscoelastic fluid obeying second-order/second-grade model as its constitutive equation with the assumption being made that the flow is symmetric and purely radial. These assumptions enabled a third-order nonlinear ODE to be obtained as the single equation governing the MHD flow of this particular fluid in flow through converging/diverging channels. With three physical boundary conditions available, Chebyshev collocation-point method was used to solve this ODE numerically. Results are presented in terms of parameters such as Reynolds number, Weissenberg number, channel half-angle, and the magnetic number. It was found that these parameters all have a profound effect on the velocity profiles in Jeffrey-Hamel flows. The effect of magnetic field was found to be more striking in that it is predicted to force fluid elements near the wall to exceed centerline velocity in converging channels and to suppress separation in diverging channels. Interestingly, the effect of the magnetic field in delaying flow separation is predicted to become more pronounced the higher the fluid's elasticity.

Keywords: Second-order/second-grade model; Converging/diverging channel; MHD flow; Separation; Weissenberg number

1. Introduction

Flow through converging/diverging channels is of paramount importance in many industrial applications [1–5]. One can mention, for example, flow through nozzles, diffusers and reducers as encountered in polymer processing operations [1–5]. Convergent/divergent flows have also been used with great success to simulate flows of dilute polymer solutions through porous media [6]. The cold-drawing operation in polymer industry also relies on flow through converging channels to improve mechanical properties of products such as plastic sheets and rods [7]. Extrusion of molten polymers through converging dies is another important application for this type of flow [8–10].

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Obviously, flow through converging/diverging channels is of importance to both Newtonian and non-Newtonian fluids alike. This can explain why the literature is so rich in relation to this kind of flow [1–19]. For Newtonian fluids further interest in this particular flow originates from the fact that it renders itself to an exact (purely radial) solution known as Jeffrey-Hamel flow [20]. This exact laminar solution has the merit that it can be used as the base flow in any instability analysis to find out the critical conditions for the occurrence of transition from laminar to turbulent flow [21]. For Newtonian fluids, further interest in this exact solution originates from the fact that it can be used to verify the validity of the Navier–Stokes equations. As a matter of fact this particular flow has been used with great success to correctly predict the critical conditions for the occurrence of flow separation in a diverging channel [20]—an important prediction from an industrial standpoint realizing the fact that separation can severely affect the performance of devices such as diffusers.

Separation is indeed a common phenomenon with diffusers and may be witnessed at any Reynolds number depending on the length and/or channel half-angle. It is undesirable by any account as it lowers the pressure recovery ratio of a diffuser and increases its head loss at the same time. To avert this phenomenon from happening, one approach is to increase the length of the diffuser and/or to decrease its subtended angle thereby weakening the adverse pressure gradient which has caused flow separation at the first place [20]. When this is not feasible, one may resort to the injection/suction of fluid particles to/from the boundary layer provided that the channel walls are porous [22]. As the last resort, one may reluctantly decide to lower the working flow rate [20]. Interestingly, in a recent work [23] it has been shown that by an increase in the elastic properties of the working fluid (say, through the use of polymeric additives) it is possible to delay flow separation in a diverging channel. As to the mechanism(s) involved, separation suppression using polymeric additives was attributed to the effect of polymer chains on the velocity and stress fields which happens to be in the right direction to prevent flow separation in a diffuser [23].

Like polymeric additives, magnetic fields are also known to affect flow kinematics in many fluid flows. As a matter of fact, after the pioneering works of Hartman and Lazarus in relation to the effect of magnetic fields on the laminar flow of viscous fluids between parallel plates [24], literature has witnessed similar effects in many other flow geometries including converging/diverging channels [25–36]. Nowadays, magnetohydrodynamic (MHD) systems are used effectively in many applications such as power generators, pumps, accelerators, electrostatic filters, and droplet purifiers, among others [37]. The question then arises as to whether the flow modification as caused by a magnetic field is of any good as far as flow separation in diverging channels is concerned. The answer to this question has recently been shown to be positive as far as Newtonian fluids are concerned [35,36]. But in relation to viscoelastic fluids, there appears to be no such an analysis in the open literature, at least to the best of our knowledge. In the present work, it will be shown that for viscoelastic fluids, too, it is possible to delay flow separation in a diverging channel provided that the magnetic field is sufficiently strong. It will also be shown that the effect of magnetic field on flow separation becomes more pronounced the higher the fluid's elasticity. To achieve these objectives, it will be shown that, quite fortunately, MHD flows of second-order/second-grade fluids through converging/diverging channels render themselves to a locally valid similarity solution.

2. Formulation of the problem

To begin with, Fig. 1 shows the geometry of interest which is seen to be a two-dimensional converging/diverging channel with subtended angle 2α. The channel walls are assumed to extend to infinity in the z-direction (i.e., perpendicular to the plane). As such, plane polar coordinates, with its origin at the apex, will be used to formulate the problem. The fluid itself is assumed to be an incompressible, electrically-conducting, viscoelastic fluid obeying the second-order or the second-grade model as its constitutive equation. As shown in Fig. 1, a uniform magnetic field of strength B is being applied on the flowing fluid in the transverse direction. It is assumed that the fluid's physical/rheological properties are not affected by the magnetic field—an assumption which naturally excludes Electro-Rheological (ER) and Magneto-Rheological fluids from our analysis [38]. It is further assumed that the flow is symmetric and purely radial. These assumptions mean that in polar coordinates there is only one non-zero velocity component, v_r, which under steady state conditions is a function of r and θ only. Having said this, it should be stressed that radial-flow assumption is a restrictive assumption even for Newtonian fluids and can easily be violated [39,40]. But, fortunately, this assumption is known to