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International Journal of Engineering Science 44 (2006) 1482–1497
International Journal of Engineering Science
www.elsevier.com/locate/ijengsci

An exact thermopiezoelasticity solution for a three-phase composite cylinder

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Received 29 May 2006; received in revised form 17 August 2006; accepted 19 August 2006
Available online 2 November 2006

Abstract

An exact solution is first derived for the stress field in a three-phase piezoelectric composite cylinder induced by a uniform heat flux together with an anti-plane shear and in-plane electric field applied at infinity. Based on the method of analytical continuation in conjunction with the alternating technique, the general expressions of the temperature and stress functions are derived explicitly in each medium of a three-phase composite cylinder. It is discovered that the stress and the electric displacement in the inclusion is always linearly proportional to the coordinate. The numerical results demonstrates that the continuity conditions at the interface are indeed satisfied and shows the effects of material mismatch on the stress and electric displacement fields.

Keywords: Analytical continuation; Alternating technique; Thermopiezoelasticity

1. Introduction

In the past few years, widespread attention has been given to piezoelectric composites and piezoelectric ceramics due to their potential applications in intelligent structure devices. Piezoelectric ceramic is composed of crystallites, grain boundaries and pores, and it may also contain many voids and conductible inclusions. When the piezoelectric ceramics are subjected to thermal and electromechanical loads in service, they can fail prematurely due to these defects during their manufacturing process. Therefore, it is important to analyze the behavior of various defects in thermal and electromechanical fields that predict the performance and integrity of these devices. Without considering thermal effect, a number of contributions concerning the inclusion problem in piezoelectric materials have appeared in the literature. These include the works of Pak [1], Kattis and Meguid [2], Honein et al. [3], Chen and Shen [4], Zhong and Meguid [5], Zhang et al. [6], Chao and Chang [7],

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0020-7225/$ - see front matter
doi:10.1016/j.ijengsci.2006.08.011

Jiang and Cheung [8], Sudak [9], Shen et al. [10,11], among others. Although the problems of inclusions problem under electromechanical loadings has been studied thoroughly, the similar problem under thermal loading conditions has rarely been reported in the literature due to the increased mathematical difficulties. Based on the modified Stroh formalism, Lu et al. [12] investigated the problem of a piezoelectric material with an elliptic cavity under a uniform heat flow. Qin et al. [13] applied the technique of conformal mapping and some identities in the Stroh formalism to treat the plane problem of thermopiezoelasticity with various holes and subject to coupled mechanical, electric and thermal loads. Wang and Shen [14] employed the complex method in assistance with the techniques of conformal mapping and analytical continuation to solve the problem of an elliptic piezoelectric inclusion embedded in an infinite plane subject to a uniform heat flow. But the aforementioned studies for non-isothermal conditions are limited to a hole or a two-phase composite. It is noted that most of the materials for engineering applications are multiphase system. In this case, solutions to both the heat conduction and piezoelasticity problems for all layers are required. These solutions are also to satisfy both the thermal and electromechanical boundary and interface conditions.

In the present paper, we consider the problem of circularly cylindrical layered media in plane thermopiezoelasticity. The layers are coaxial cylinders of annular cross-sections with arbitrary radii and different material properties. The proposed method is based on the technique of analytical continuation that is alternatively applied across the two concentric interfaces in order to derive the solution from the corresponding homogeneous solution.

Following the brief introduction, basic equations concerning the thermopiezoelasticity are summarized in Section 2. The general forms of the complex potentials of the temperature and stress functions are derived in Sections 3 and 4, respectively. Numerical results and discussion are given in Section 5. Finally, Section 6 concludes the article.

2. Formulation of the problem

Consider a piezoelectric composite composed of three dissimilar materials bonded along two concentric interfaces as shown in Fig. 1. Each layer of the composite is assumed to be transversely isotropic with respect to the longitudinal direction. A far-field anti-plane shear and in-plane electric field together with a uniform heat flux causes a thermal stress distribution as a result of the different thermopiezoelastic properties of a three-phase cylindrically concentric solid.

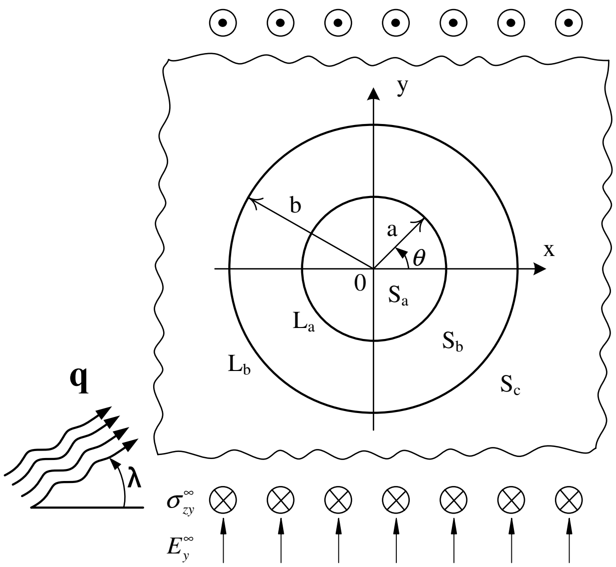

Fig. 1. A three-phase cylinder subjected to a remote uniform heat flux and electromechanical loads.