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Unilateral contact with adhesion and friction between two hyperelastic bodies

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Abstract

In the present paper, we consider a thermodynamic model using the contact kinematics developed by A. Curnier, Q.C. He and J.J. Télega [C. R. Acad. Sci. Paris Sér. II 314 (1992) 1] involving unilateral contact, adhesion and Coulomb friction between two homogeneous, isotropic and hyperelastic bodies. Adhesion is described by an internal state variable β^{φ} introduced by M. Frémond [C. R. Acad. Sci. Paris Sér. II 295 (1982) 913; J. Theor. Appl. Mech. 6 (1987) 383]. Taking the case of contact between a hyperelastic solid and a plane support, we formulate the associated boundary value problem as a minimization problem when no friction is involved. When the intensity of the adhesion obeys a 'static' law, we obtain an existence result for this problem. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In this paper, we propose a thermodynamic model considering both unilateral contact, adhesion and friction between two elastic bodies in the framework of finite deformations [4]. Many interface debonding models have been developed in the framework of small displacements and small deformations. Our model deals with the evolution of the decohesion at the interface between two bodies. Cohesive forces at the interface may act in both the normal and tangential directions. Some of these models take into account unilateral conditions of non-penetration and friction

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between the bodies. This is the case of the model proposed by Cangémi [5] and Raous et al. [20,21]. With our formulation, the interface debonding models are extended to include the case of finite deformations, decohesion, friction and unilateral contact. According to Frémond [10,11], the adhesion is characterized by its intensity β^{φ}, which can vary from 1 (perfect adhesion) to 0 (no adhesion).

The thermodynamic bases of the model are presented in part 2. We begin by giving the contact kinematics in the framework of finite deformations. Using an approach proposed by Curnier et al. [7], we define the gap vector via the orthogonal projection of a point belonging to one of the candidate potential contact surfaces onto the other one. Then we define an objective relative contact velocity of one surface with respect to the other. This local approach is suitable for defining the impenetrability condition via a signed gap vector. Another approach, proposed by Ciarlet and Nečas [6], consists in adopting an overall formulation, assuming a unilateral contact condition under the constraint to stay within a set. Since this does not seem to be very suitable for numerical investigations, we adopted the local approach. The thermodynamic bases of the model are developed and we describe only the contact behavior of one of the two surfaces, as we are able to deduce the behavior of the other one from the extended action–reaction principle [7]. When the free energy density and the pseudo-potential of dissipation are suitably chosen, the complete boundary value problem is obtained.

Finally, in the particular case of contact with a plane rigid support, we propose a variational formulation of the problem with respect to the reference configuration. When the adhesive contact is assumed to be frictionless, the corresponding variational problem can be reduced to a minimization problem, for which an existence result can be obtained if the intensity of the adhesion obeys a ‘static’ law [2,6,7].

2. A thermodynamic model for unilateral contact, adhesion and friction

2.1. Contact kinematics

Throughout this paper, the variable α will be assumed to be 1 and 2. We consider two solids $\mathscr{B}^\alpha$ occupying, in some reference configurations, two domains $\Omega^\alpha \in \mathbb{R}^3$ with their boundaries denoted by Γ^α and let $\mathbf{n}^\alpha$ be the corresponding outward normal unit vector. We will identify each particle of $\mathscr{B}^\alpha$ with its position vector in Ω^α denoted by $\mathbf{X}^\alpha$. Motions of bodies are defined by the locally invertible and orientation-preserving mappings φ^α such that for all $t \in [0, T]$

$$\varphi^\alpha(\cdot, t) : \bar{\Omega}^\alpha \to \mathbb{R}^3.$$

The sets $\Omega^{\varphi^\alpha} = \varphi^\alpha(\Omega^\alpha, t)$ are the deformed counterparts of Ω^α, and $\mathbf{n}^{\varphi^\alpha}$ is the outward normal unit vector to $\partial\Omega^{\varphi^\alpha}$. Let us denote by $\mathbf{x}^{\varphi^\alpha} = \varphi^\alpha(\mathbf{X}^\alpha, t)$ the position vector in the deformed configuration. Finally, we assume that the boundaries are regular enough, and each of them is decomposed into three disjoint open parts, denoted by $\Gamma^\alpha_{\mathrm{D}}, \Gamma^\alpha_{\mathrm{T}}, \Gamma^\alpha_{\mathrm{C}}, \Gamma^{\varphi^\alpha}_{\mathrm{D}} = \varphi^\alpha(\Gamma^\alpha_{\mathrm{D}}, t)$, $\Gamma^{\varphi^\alpha}_{\mathrm{T}} = \varphi^\alpha(\Gamma^\alpha_{\mathrm{T}}, t)$ and $\Gamma^{\varphi^\alpha}_{\mathrm{C}} = \varphi^\alpha(\Gamma^\alpha_{\mathrm{C}}, t)$, respectively. We assume that the deformations are prescribed on $\Gamma^\alpha_{\mathrm{D}}$ and $\Gamma^{\varphi^\alpha}_{\mathrm{D}}$ in both configurations, that the surface density forces $\mathbf{T}^\alpha, \mathbf{T}^{\varphi^\alpha}$ on $\Gamma^\alpha_{\mathrm{T}}, \Gamma^{\varphi^\alpha}_{\mathrm{T}}$ are, respectively, applied and that $\Gamma^\alpha_{\mathrm{C}}, \Gamma^{\varphi^\alpha}_{\mathrm{C}}$ are the potential contact surfaces in the two configurations, where the bodies can