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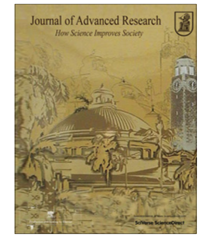

ORIGINAL ARTICLE

Control and switching synchronization of fractional order chaotic systems using active control technique

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A B S T R A C T

This paper discusses the continuous effect of the fractional order parameter of the Lü system where the system response starts stable, passing by chaotic behavior then reaching periodic response as the fractional-order increases. In addition, this paper presents the concept of synchronization of different fractional order chaotic systems using active control technique. Four different synchronization cases are introduced based on the switching parameters. Also, the static and dynamic synchronizations can be obtained when the switching parameters are functions of time. The nonstandard finite difference method is used for the numerical solution of the fractional order master and slave systems. Many numeric simulations are presented to validate the concept for different fractional order parameters.

Introduction

During the last few decades, fractional calculus has become a powerful tool in describing the dynamics of complex systems which appear frequently in several branches of science and engineering. Therefore fractional differential equations and their numerical techniques find numerous applications in the field of viscoelasticity, robotics, feedback amplifiers, electrical circuits, control theory, electro analytical chemistry, fractional multi-poles, chemistry and biological sciences [1–12].

The chaotic dynamics of fractional order systems began to attract a great deal of attention in recent years due to the ease of their electronic implementations as discussed before [13,14]. Due to the very high sensitivity of these chaotic systems which is required for many applications, there was a need to discuss the coupling of two or more dissipative chaotic systems which is known as synchronization. Chaotic synchronization has been applied in many different fields, such as biological and physical systems, structural engineering, ecological models [15,16].

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Pecora and Carroll [15] were the first to introduce the concept of synchronization of two systems with different initial conditions. Many chaotic synchronization schemes have also been introduced during the last decade such as adaptive control, time delay feedback approach [17,18], nonlinear feedback synchronization, and active control [19]. However, most of these methods have been tested for two identical chaotic systems. When Ho and Hung [19] presented and applied the concept of active control method on the synchronization of chaotic systems, many recent papers investigated this technique for different systems and in different applications [20,21]. The synchronization of three chaotic fractional order Lorenz systems with bidirectional coupling in addition to the chaos synchronization of two identical systems via linear control was investigated [22,23]. Moreover, two different fractional order chaotic systems can be synchronized using active control [24]. The hyper-chaotic synchronization of the fractional order Rössler system which exists when its order is as low as 3.8 was shown by Yua and Lib [25]. Recently the consistency for the improvement of models based on fractional order differential structure has increased in the research of dynamical systems [26]. In addition, many researchers have studied the control of systems in different applications [27,28], in addition to the circuit and electromagnetic theories as shown by others [3,4,10–12,29].

Several analytical and numerical methods have been proposed to solve the fractional order differential equations for example the nonstandard finite difference schemes (NSFDs), developed by Mickens [30,31] have shown great potential in recent applications [32,33].

There are two aims for this paper, the first aim is to study the proper fractional order range which exhibits chaotic behavior for the Lü system. More than thirty cases are investigated for different orders and changing only a single system parameter. Stable, periodic and chaotic responses are shown for each system parameter but with different fractional order ranges. The second aim is to discuss the active technique for the synchronization of two different fractional order chaotic systems and using two on/off switches. Based on the proposed technique, static and dynamic synchronization can be obtained in four different cases. The numerical solutions of the fractional order for the master, slave and error systems are computed using NSFD.

In ‘Fundamentals of fractional order’ the basic fundamentals of the fractional order will be discussed. ‘Grünwald–Letnikov approximation’ will introduce the effect of the fractional order parameter of the fractional Lü system on the output response. The concept of active control using two on/off switches for the synchronization between two different chaotic systems will be proposed in ‘Non-standard Discretization’. Four different static and dynamic synchronization cases will be introduced in ‘Effect of the fractional order parameter on the Lü system response’ based on changing the switching parameters with time. Finally, conclusions are drawn in the last section.

Fundamentals of fractional order

Although the concept of the fractional calculus was discussed in the same time interval of integer order calculus, the complexity and the lack of applications postponed its progress till a few decades ago. Recently, most of the dynamical systems based on the integer-order calculus have been modified into the fractional order domain due to the extra degrees of freedom and the flexibility which can be used to precisely fit the experimental data much better than the integer-order modeling. For example, new fundamentals have been investigated in the fractional order domain for the first time and do not exist in the integer-order systems such as those presented in [4,6,9–12]. The Caputo fractional derivative of order α of a continuous function $f: R^+ \to R$ is defined as follows:

$$D^\alpha f(t) \equiv \frac{d^\alpha f(t)}{dt^\alpha} = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau & m-1 < \alpha < m \\ \frac{d^m}{dt^m} f(t) & \alpha = m \end{cases} \tag{1}$$

where m is the first integer greater than α, and $\Gamma(\cdot)$ is the Gamma function and is defined by:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \quad \Gamma(z+1) = z\Gamma(z) \tag{2}$$

In this section, some basic definitions and properties of the fractional calculus theory and nonstandard discretization are discussed.

Grünwald–Letnikov approximation

The Grünwald–Letnikov method of approximation for the one-dimensional fractional derivative is as follows [34]:

$$D^\alpha x(t) = f(t, x) \tag{3}$$

$$D^\alpha x(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{t/h} (-1)^j \binom{\alpha}{j} x(t - jh) \tag{4}$$

where $\alpha > 0$, D^α denotes the fractional derivative. $N = [t/h]$, and h is the step size. Therefore, Eq. (3) is discretized as follows:

$$\sum_{j=0}^{n+1} c_j^\alpha x(t - jh) = f(t_n, x(t_n)), \quad n = 1, 2, 3, \ldots, \tag{5}$$

where $t_n = nh$ and c_j^α are the Grünwald–Letnikov coefficients defined as:

$$C_j^\alpha = \left(1 - \frac{1+\alpha}{j}\right) c_{j-12}^\alpha, \quad \text{and} \quad c_0^\alpha = h^{-\alpha}, \quad j = 1, 2, 3, \ldots \tag{6}$$

Nonstandard discretization

The nonstandard discretization technique is a general scheme where we replace the step size h by a function $\varphi(h)$. By applying this technique and using the Grünwald–Letnikov discretization method, it yields the following relations

$$x_{n+1} = \frac{-\sum_{j=1}^{n+1} c_j^\alpha x_{n+1-j} + f_1(t_{n+1}, x_{n+1})}{c_0^{\alpha_1}} \tag{7}$$

where $c_0^{\alpha_1} = (\varphi_1(h))^{-1}$ are functions of the step size $h = \Delta t$, with the following properties:

$$\varphi_1(h) = h + O(h^2), \quad \text{where } h \to 0 \tag{8}$$