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ORIGINAL ARTICLE

Vibration analysis of structural elements using differential quadrature method

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Abstract The method of differential quadrature is employed to analyze the free vibration of a cracked cantilever beam resting on elastic foundation. The beam is made of a functionally graded material and rests on a Winkler–Pasternak foundation. The crack action is simulated by a line spring model. Also, the differential quadrature method with a geometric mapping are applied to study the free vibration of irregular plates. The obtained results agreed with the previous studies in the literature. Further, a parametric study is introduced to investigate the effects of geometric and elastic characteristics of the problem on the natural frequencies.

Introduction

In recent years, differential quadrature method (DQM) has become increasingly popular in the numerical solution of initial and boundary value problems. The advantages of this method lie in its easy use and flexibility with regard to arbitrary grid spacing. Also, DQM method can yield accurate results with relatively much fewer grid points compared with the previous numerical techniques such as the finite element and finite difference methods. The present work aims to realize the ability of DQM to solve two complicated problems. The first one concerns with the free vibration of elastically supported cracked beams and the second problem concerns with the free vibration of irregular plates.

In general, there are two approaches to analyze the free vibration of the cracked beams. The first one employees the variational principles through a continuous model, see for example, those in [1,2]. The second approach employees the line spring models (LSMs) to simulate existence of the cracks. Shen and Pierre [3] analyzed the free vibration of beams with pairs of symmetric open cracks. Yokoyama and Chen [4] examined the vibration characteristics of a Bernoulli–Euler beam with an edge crack. Qian et al. [5] explained the dynamic behavior and crack detection of a beam with a crack by using the finite element method. Gudmundson [6,7] discussed the dynamic model for beams with cross sectional crack and predicted the changes in resonance frequencies of a structure

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resulting from the crack. Rizos et al. [8] analyzed the cracked structures by measuring the modal characteristics. All of these works [3–8] concerned with the cracked beams made of an isotropic materials.

Also, there are several publications concerned with the vibration analysis of plates. Leissa [9] derived exact solutions for the free vibration problems of the rectangular plates. Xiang et al. [10] used Ritz method to analyze the vibration of rectangular Mindlin plates resting on elastic edge supports. More recently, DQM is extensively applied for solving vibration problems. Bert and Malik [11] introduced a review on the early stages of the method development and its applications. Also, they [12] made the first attempt to apply DQM for vibration analysis of irregular plates. Liew et al. [13] and Han and Liew [14] also used a similar approach to analyze irregular quadrilateral thick plates. Lam [15] introduced a mapping technique to apply the DQ method to conduction, torsion, and heat flow problems with arbitrary geometries.

Functionally graded materials (FGMs), a novel class of macroscopically nonhomogeneous composites with spatially continuous material properties, have attracted considerable research efforts over the past few years due to their increasing applications in many engineering fields. Numerous studies have been conducted on FGM beams and plates, dealing with a variety of subjects such as thermal elasticity [16,17], fracture mechanics [18,19], and vibration analysis [20–25].

The present work aims to extend the applications of DQM to solve two difficult problems. The first one concerns with the free vibration of an elastically supported cracked beam. The beam is made of a FGM and rests on a Winkler–Pasternak foundation. The line spring model is employed to simulate the crack actions. In the second problem, the DQM with a mapping technique are applied to analyze the free vibration of irregular plates. The obtained results are agreed with the previous similar ones. Further, a parametric study is introduced to explain the effects of elastic and geometric characteristics of the problem on the values of natural frequencies.

Methodology

Free vibration analysis of cracked beams

Consider an elastically cantilever beam of length L and thickness h, containing an edge crack of depth a located at a distance L_1 from the left end, as shown in Fig. 1. The beam is made of a FGM, such that shear modulus, Young's modulus, and mass density of the beam vary in the thickness direction only as follows [28]:

$$\mu(z) = \mu_0 e^{\alpha z}, \quad E(z) = E_0 e^{\alpha z}, \quad \rho(z) = \rho_0 e^{\alpha z}, \tag{1}$$

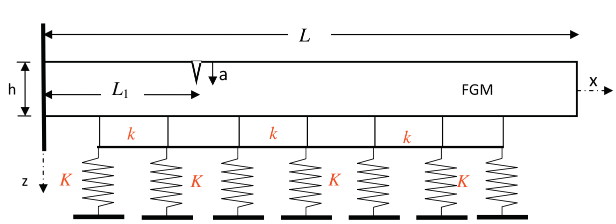

Fig. 1 Cracked functionally graded beam on a Winkler–Pasternak foundation.

where μ_0, E_0, and ρ_0 are shear modulus, Young's modulus, and mass density at the mid-plane, ($z = 0$), of the beam. α is a constant characterizing the beam material grading, $\alpha = \ln(E_2/E_1)/h$, where E_1 and E_2 are the values of Young's modulus at the lower and upper beam surfaces, respectively.

It is assumed that the crack is always open and its surfaces are free of traction, such that the beam can be treated as a two sub-beams connected by an elastic rotational spring at the cracked section which has no mass and no length. The bending stiffness of the cracked section, K_T, is related to the flexibility G by:

$$K_T = \frac{1}{G} \tag{2}$$

Referring to Broek's approximation [26], one can find that flexibility is governed by:

$$\frac{dG}{da} = \frac{2w(1 - \nu^2)K_I^2}{E(z)M_1^2}, \tag{3}$$

where is Poisson's ratio. M_1 is bending moment at the cracked section. K_I is mode I stress intensity factor, which can be obtained as a special case of the results introduced by Erdogan and Wu [27].

The governing equations, for the prescribed cracked FGM beam, can be written as [28]:

$$A_{11}\frac{\partial^2 u_i}{\partial X^2} - B_{11}\frac{\partial^3 w_i}{\partial X^3} = 0 \quad (i = 1, 2), \tag{4}$$

$$\left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)\frac{\partial^4 w_i}{\partial X^4} + Kw_i - k\frac{\partial^2 w_i}{\partial X^2} + I_1\frac{\partial^2 w_i}{\partial t^2} = 0, \quad (i = 1, 2), \tag{5}$$

where the subscript $i = 1$ stands for the left sub-beam in ($0 \leqslant X \leqslant L_1$), while $i = 2$ holds true for the right sub-beam occupying ($L_1 \leqslant X \leqslant L$), see Fig. 1.

w_i, u_i are the components of displacement vector in the directions of z and x, respectively. t is time. K, k are elastic and shear modules of the foundation reaction, respectively.

$$(A_{11}, B_{11}, D_{11}) = \int_{-h/2}^{h/2} \frac{E(z)}{1 - \nu^2}(1, z, z^2)dz, \quad I_1 = \int_{-h/2}^{h/2} \rho(z)dz \tag{6}$$

The normal force $\overline{N}_i(X, t)$, bending moment $\overline{M}_i(X, t)$, and transverse shear force $\overline{Q}_i(X, t)$ are related to the displacement components as follows [28]:

$$\overline{N}_i(X, t) = A_{11}\frac{\partial u_i}{\partial X} - B_{11}\frac{\partial^2 w_i}{\partial X^2}, \quad (i = 1, 2), \tag{7}$$

$$\overline{M}_i(X, t) = B_{11}\frac{\partial u_i}{\partial X} - D_{11}\frac{\partial^2 w_i}{\partial X^2}, \quad (i = 1, 2), \tag{8}$$

$$\overline{Q}_i(X, t) = B_{11}\frac{\partial^2 u_i}{\partial X^2} - D_{11}\frac{\partial^3 w_i}{\partial X^3}, \quad (i = 1, 2). \tag{9}$$

Let x be a normalized parameter defined as:

$$x = \begin{cases} X/L_1 & i = 1 \\ (X - L_1)/L_2 & i = 2 \end{cases}, \quad L_2 = L - L_1.$$

Also for free vibration analysis, let the prescribed field quantities can be expressed as:

$$\begin{aligned} &u_i(x, t) = U_i(x)\sin\omega t, \quad w_i(x, t) = W_i(x)\sin\omega t, \\ &\overline{N}_i(x, t) = N_i(x)\sin\omega t, \quad \overline{M}_i(x, t) = M_i(x)\sin\omega t, \\ &\overline{Q}_i(x, t) = Q_i(x)\sin\omega t, \quad (i = 1, 2). \end{aligned} \tag{10}$$

where ω is the natural frequency of the cracked FG beam.