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ORIGINAL ARTICLE

Effect of Hall current and thermal radiation on heat and mass transfer of a chemically reacting MHD flow of a micropolar fluid through a porous medium

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Abstract Heat and mass transfer effects on an unsteady flow of a chemically reacting micropolar fluid over an infinite vertical porous plate through a porous medium in the presence of a transverse magnetic field with Hall effect and thermal radiation are studied. The governing system of partial differential equations is transformed to dimensionless equations using dimensionless variables. The dimensionless equations are then solved analytically using the perturbation technique to obtain the expressions for velocity, microrotation, temperature and concentration. With the help of graphs, the effects of the various important parameters entering into the problem on the velocity, microrotation, temperature and concentration fields within the boundary layer are discussed. Also the effects of the pertinent parameters on the skin friction coefficient and rates of heat and mass transfer in terms of the Nusselt number and Sherwood number are presented numerically in a tabular form. The results show that the observed parameters have a significant influence on the flow, heat and mass transfer.

1. Introduction

In many transport processes existing in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusion of heat and chemical species, the study of such processes is useful for improving a number of chemical technologies such as polymer production, enhanced oil recovery, underground energy transport, manufacturing of ceramics and food processing. Heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as drying of porous solids, thermal insulations, and cooling of nuclear reactors. At high operating temperature, radiation effects can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of reliable equipments, nuclear plants, gas turbines and

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various propulsion devices or aircraft, missiles, satellites and space vehicles. Micropolar fluids are those consisting of randomly oriented particles suspended in a viscous medium, which can undergo a rotation that can affect the hydrodynamics of the flow, making it a distinctly non- Newtonian fluid. They constitute an important branch of non-Newtonian fluid dynamics where microrotation effects as well as microinertia are exhibited. The theory of micropolar fluids originally developed by Erigen, 1966 has been a popular field of research in recent years. Eringen's theory has provided a good model for studying a number of complicated fluids, such as colloidal fluids, polymeric fluids and blood. Micropolar fluid flow induced by the simultaneous action of buoyancy forces is of great interest in nature and in many industrial applications as drying processes, solidification of binary alloy as well as in astrophysics, geophysics and oceanography.

When the strength of the magnetic field is strong, one cannot neglect the effect of Hall current. It is of considerable importance and interest to study how the results of the hydrodynamical problems get modified by the effect of Hall currents. Hall currents give rise to a cross flow making the flow three dimensional. Several authors ((Eldabe and ouat, 2006); (Keelson and Desseaux, 2001); (Mahmoud, 2007); (Magdy, 2005); (Modather et al., 2009); (Nadeem et al., 2010); (Omokhuale et al., 2012); (Patil and Kulkarni, 2008); (Rehbi et al., 2007); (Roslinda et al., 2008); (Srinivasachanya and Ramreddy, 2011); (Sunil et al., 2006)) studied MHD flow of a micropolar fluid. Rakesh and Khem (2011) studied the effect of slip conditions and Hall current on unsteady MHD flow of a viscoelastic fluid past an infinite vertical porous plate through a porous medium.

We extended the work of Rakesh and Khem, 2011 by incorporating angular momentum and concentration equations with thermal radiation and chemical reaction terms in the absence of viscoelastic term to study Hall current and thermal radiation on heat and mass transfer of unsteady MHD flow of a chemically reacting micropolar fluid through a porous medium. The governing equations are solved analytically using the perturbation method and the effect of various physical parameters is discussed numerically and graphically.

2. Mathematical Formulation

We consider the unsteady flow of a viscous incompressible and electrically conducting micropolar fluid over an infinite vertical porous plate, subjected to a constant transverse magnetic field B_0 in the presence of thermal and concentration buoyancy effects.

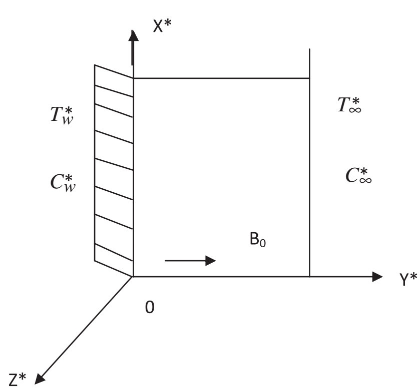

Figure 1 Physical mode.

The induced magnetic field is assumed to be negligible compared to the applied magnetic field. The x^*-axis is taken along the planar surface in the upward direction and the y^*-axis is taken to be normal to it as shown in Fig. 1. Due to the infinite plane surface assumption, the flow variables are functions of y^* and the t^* only. The plate is subjected to a constant suction velocity V_0.

The governing equations of flow under the usual Boussinesq approximation are given by

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = (v + v_r)\frac{\partial^2 u^*}{\partial y^{*2}} + v_r\frac{\partial N_1^*}{\partial y^*} + g\beta_T(T^* - T_\infty^*) + g\beta_c(C^* - C_\infty^*) - \sigma\frac{B_0^2(u^* + mw^*)}{\rho(1+m^2)} - \frac{v}{K^*}u^* \tag{2}$$

$$\frac{\partial w^*}{\partial t^*} + v^*\frac{\partial w^*}{\partial y^*} = (v + v_r)\frac{\partial^2 w^*}{\partial y^{*2}} - v_r\frac{\partial N_2^*}{\partial y^*} - \sigma\frac{B_0^2(w^* - mu^*)}{\rho(1+m^2)} - \frac{v}{K^*}W^* \tag{3}$$

$$\rho J^*\left(\frac{\partial N_1^*}{\partial t^*} + v^*\frac{\partial N_1^*}{\partial y^*}\right) = \gamma\frac{\partial^2 N_1^*}{\partial y^{*2}} \tag{4}$$

$$\rho J^*\left(\frac{\partial N_2^*}{\partial t^*} + v^*\frac{\partial N_2^*}{\partial y^*}\right) = \gamma\frac{\partial^2 N_2^*}{\partial y^{*2}} \tag{5}$$

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} \tag{6}$$

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_c^*(C^* - C_\infty^*) \tag{7}$$

The appropriate boundary conditions for the problem are

$$u^* = L^*\left(\frac{\partial u^*}{\partial y^*}\right), v^* = 0, w^* = L^*\left(\frac{\partial u^*}{\partial y^*}\right),$$
$$N_1^* = -n\frac{\partial u^*}{\partial y^*}, N_2^* = n\frac{\partial w^*}{\partial y^*} T^* = T_\infty^* + (T_w^* - T_\infty^*)e^{i\omega^* t^*}$$
$$C^* = C_\infty^* + (C_w^* - C_\infty^*)e^{i\omega^* t^*} \text{ at } y^* = 0$$
$$u^* \to 0, v^* \to 0, w^* \to 0, N_1^* \to 0, N_2^* \to 0 T^* \to T_\infty^*,$$
$$C^* \to C_\infty^* \text{ at } y^* \to \infty \tag{8}$$

where u^*, v^* and w^* are velocity components along x^*, y^* and z^*-axis respectively, N_1^* and N_2^* are microrotation components along x^* and z^*-axis respectively, v is the Kinematic viscosity, v_r is the Kinematic micro-rotation viscosity, q_r is the the radiative heat flux, g is the acceleration due to gravity, βt and βc are the coefficients of thermal expansion and concentration expansion respectively, T^* is the dimensional temperature of the fluid, T_w^* and T_∞^* denote the temperature at the plate and temperature far away from the plate respectively, C^* is the dimensional concentration of the solute, C_w^* and C_∞^* are concentration of the solute at the plate and concentration of the solute far from the plate respectively, K^* is the permeability of the porous medium, k is the thermal conductivity of the medium, ρ is the density of the fluid, j^* is the micro-inertia density or micro-inertia per unit mass, γ is the spin gradient viscosity, L^* is the characteristic length, ω^* is the dimensional frequency of oscillation, σ is the electrical conductivity, m is