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SHORT COMMUNICATION

Effect of heat radiation in a Walter's liquid B fluid over a stretching sheet with non-uniform heat source/sink and elastic deformation

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Viscous dissipation

Abstract In this present article heat transfer in a Walter's liquid B fluid over an impermeable stretching sheet with non-uniform heat source/sink, elastic deformation and radiation are reported. The basic boundary layer equations for momentum and heat transfer, which are non-linear partial differential equations, are converted into non-linear ordinary differential equations by means of similarity transformation. The dimensionless governing equations for this investigation are solved analytically using hyper geometric functions. The results are carried out for prescribed surface temperature (PST) and prescribed power law surface heat flux (PHF). The effects of viscous dissipation, Prandtl number, Eckert number, heat source/sink parameter with elastic deformation and radiation are shown in the several plots and addressed.

1. Introduction

The thermal radiation may be quite significant at high operating temperatures in engineering processes, under many non-isothermal situations and in situations where convective heat transfer coefficients are small. In polymer processing industry, if the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then thermal radiation effect might play an important role in controlling the heat transfer process. The knowledge of radiation heat transfer in the system can perhaps lead to a desired quality of the final product. In view of this, Seddeek and Abdelmeguid (2006) studied the effect of thermal radiation on the heat transfer of a Newtonian fluid having temperature dependent diffusivity past stretching surface with variable heat flux. Bataller (2007) investigated the effect of thermal radiation on the heat transfer in a boundary layer fluid flow over a stretching sheet with internal heat source/sink. Abdul Hakeem and Sathiyanathan (2009) investigated the effect of thermal radiation for an oscillatory flow analytically. Recently, Bhattacharyya and Layek (2011) studied the effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation.

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Another important aspect, that is, heat transfer processes play an important role in all such theoretical studies. This is due to the fact that the rate of cooling influences a lot to the quality of the final product with desired characteristics. Abo-Eldahab Emad and El Aziz Mohamed (2004) investigated heat transfer considering non-uniform heat source/sink. Abel and Nandeppanavar Mahantesh (2009) studied the MHD visco-elastic flow over a stretching sheet with non uniform heat source/sink.

Pillai et al. (2004) analyzed the effects of work done by deformation in viscoelastic fluid in porous media with uniform heat source. Cortell (2006) have investigated the effects of elastic deformation on MHD flow of viscoelastic fluid. Liu (2004) studied the effects of viscous dissipation, work done by deformation and uniform heat source on heat transfer in second grade fluid. Nandeppanavar Mahantesh et al. (2010) have investigated the heat transfer in a viscoelastic fluid over an impermeable stretching sheet with non-uniform heat source/sink and elastic deformation.

Keeping this in mind, we investigated the effect of heat radiation in a Walter's liquid B fluid flow over an impermeable stretching sheet with non-uniform heat source/sink and elastic deformation.

2. Mathematical formulation

Consider a steady, laminar and two dimensional flow of an incompressible Walter's liquid B fluid past a flat sheet coinciding with a plane $y = 0$ and the flow being confined to $y > 0$. The flow is generated due to the stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along the x-axis. Keeping the origin fixed, the sheet is then stretched with a speed varying linearly with the distance from the slit. We take x-axis along the surface, y-axis being normal to it and u and v are the fluid tangential velocity and normal velocity, respectively as shown in Fig. 1.

The equations governing the problem under consideration are given by

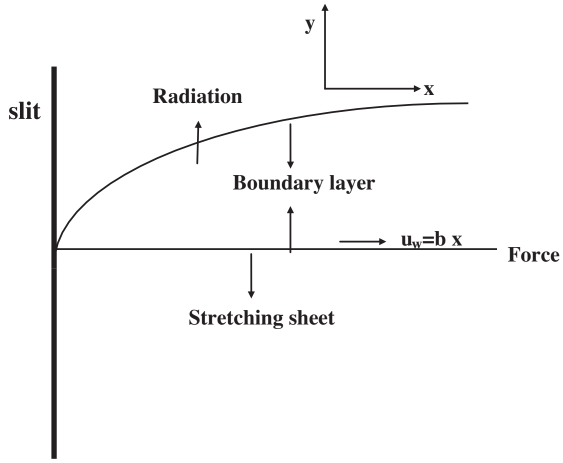

Figure 1 Boundary layer over an impermeable linear stretching sheet.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2} - k_o\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}, \tag{2}$$

where v is the kinematic viscosity, k_0 is the elastic parameter.

The boundary conditions for the velocity fields are of the form.

$$\begin{aligned} &u = bx, \quad v = 0 \quad \text{at} \quad y = 0,\\ &u \to 0 \quad \text{as} \quad y \to \infty, \end{aligned} \tag{3}$$

where b is the stretching rate.

We define the following new variables

$$u = bxf_\eta(\eta), \quad v = -(bv)^{\frac{1}{2}}f(\eta), \quad \eta = \left(\frac{b}{v}\right)^{\frac{1}{2}}y. \tag{4}$$

Using (4), Eq. (1) is trivially satisfied and Eqs. (2) and (3) takes the form:

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1\left\{2f_\eta f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} - f_{\eta\eta}^2\right\}. \tag{5}$$

With corresponding boundary conditions

$$\begin{aligned} &f_\eta(\eta) = 1, \quad f(\eta) = 0 \quad \text{at} \quad \eta = 0,\\ &f_\eta(\eta) \to 0, \ \text{as} \ \eta \to \infty. \end{aligned} \tag{6}$$

Here the subscript η denotes differentiation with respect to η, $k_1 = \frac{k_0 b}{v}$ is the viscoelastic parameter.

Here boundary conditions are one less in number required to solve the flow problem uniquely. Following Rajagobal et al. (1987), the solution of Eq. (5) with boundary conditions (6) can be written in the form:

$$f(\eta) = \frac{1-e^{-\alpha\eta}}{\alpha}, \tag{7}$$

where $\alpha = \sqrt{\frac{1}{1-k_1}}$. Obviously $0 < k_1 < 1$.

Using the solution (7) in Eq. (4), the velocity components obtained in the form

$$u = bxe^{-\alpha\eta}, \ \text{and} \ v = -\sqrt{bv}\left\{\frac{1-e^{-\alpha\eta}}{\alpha}\right\}. \tag{8}$$

The wall shearing stress on the surface of the stretching sheet is given by

$$\tau_w = \left[v\frac{\partial u}{\partial y} - k_0\left(u\frac{\partial^2 u}{\partial x\partial y} - 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right)\right]_{y=0}. \tag{9}$$

The local skin-friction coefficient or the frictional drag is given by

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_w^2} = 2Re_x^{-1/2}(1-3k_1)f_{\eta\eta}(0). \tag{10}$$

3. Heat transfer analysis

The governing thermal boundary layer equation in the presence of viscous dissipation, elastic deformation non uniform internal heat source/sink, radiation for considered two - dimensional flow problem is given by