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ORIGINAL ARTICLE

Convective transport in a porous medium layer saturated with a Maxwell nanofluid

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Abstract A linear and weakly non-linear stability analys is has been carried out to study the onset of convection in a horizontal layer of a porous medium saturated with a Maxwell nanofluid. To simulate the momentum equation in porous media, a modified Darcy–Maxwell nanofluid model incorporating the effects of Brownian motion and thermophoresis has been used. A Galerkin method has been employed to investigate the stationary and oscillatory convections; the stability boundaries for these cases are approximated by simple and useful analytical expressions. The stability of the system is investigated by varying various parameters viz., nanoparticle concentration Rayleigh number, Lewis number, modified diffusivity ratio, porosity, thermal capacity ratio, viscosity ratio, conductivity ratio, Vadász number and relaxation parameter. A representation of Fourier series method has been used to study the heat and mass transport on the non-linear stability analysis. The effect of transient heat and mass transport on various parameters is also studied. It is found that for stationary convection Lewis number, viscosity ratio and conductivity ratio have a stabilizing effect while nanoparticle concentration Rayleigh number *Rn* destabilizes the system. For oscillatory convection we observe that the conductivity ratio stabilizes the system whereas nanoparticle concentration Rayleigh number, Lewis number, Vadász number and relaxation parameter destabilize the system. The viscosity ratio increases the thermal Rayleigh number for oscillatory convection initially thus delaying the onset of convection and later decreases thus advancing the onset of convection hence showing a dual effect. For steady finite amplitude motions, the heat and mass transport decreases with an increase in the values of nanoparticle concentration Rayleigh number, Lewis number, viscosity ratio and conductivity ratio. The mass transport increases with an increase in Vadász number and relaxation parameter. We also study the effect

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of time on transient Nusselt number and Sherwood number which are found to be oscillatory when time is small. However, when time becomes very large both the transient Nusselt and Sherwood values approach to their steady state values.

Nomenclature

D_B	Brownian diffusion coefficient (m^2/s)
D_T	thermophoretic diffusion coefficient (m^2/s)
H	dimensional layer depth (m)
k	thermal conductivity of the nanofluid (W/m K)
k_m	overall thermal conductivity of the porous medium saturated by the nanofluid (W/m K)
K	permeability (m^2)
Ln	Lewis number
N_A	modified diffusivity ratio
N_B	modified particle-density increment
p^*	pressure (Pa)
p	dimensionless pressure, $(p^*K)/(\mu\alpha_f)$
λ	non dimensional relaxation time
Va	Vadász number
γ_a	non dimensional acceleration coefficient
Ra_T	thermal Rayleigh – Darcy number
Rm	basic-density Rayleigh number
Rn	concentration Rayleigh number
t^*	time (s)
t	dimensionless time, $(t^*\alpha_f)/H^2$
T^*	nanofluid temperature (K)
T	dimensionless temperature, $\frac{T^*-T_c^*}{T_h^*-T_c^*}$
T_c^*	temperature at the upper wall (K)
T_h^*	temperature at the lower wall (K)
(u, v, w)	dimensionless Darcy velocity components $(u^*, v^*, w^*)H/\alpha_m$ (m/s)
$\mathbf{v}$	nanofluid velocity (m/s)
(x, y, z)	dimensionless Cartesian co-ordinate $(x^*, y^*, z^*)/H$; z is the vertically upward co-ordinate
(x^*, y^*, z^*)	cartesian co-ordinates

Greek symbols

α_f	thermal diffusivity of the fluid, (m/s^2)
β	thermal volumetric coefficient (K^{-1})
ν	viscosity variation parameter
ε	porosity
η	conductivity variation parameter
μ	viscosity of the fluid
ρ	fluid density
ρ_p	nanoparticle mass density
σ	thermal capacity ratio
ϕ^*	nanoparticle volume fraction
ϕ	relative nanoparticle volume fraction, $\frac{\phi^*-\phi_0^*}{\phi_1^*-\phi_0^*}$

Superscripts

*	dimensional variable
′	perturbed variable
St	stationary
Osc	oscillatory

Subscripts

b	basic solution
f	fluid
p	particle

1. Introduction

Nanofluids are termed as the suspensions of nanoparticles in liquids. In the industrial applications such thermal storage, thermal management technologies, technologies that employ nanofluids require sustainable energy. Using nanofluids the transport properties of heat transfer can be enhanced. For example Xuan and Li (2003) reported that there is a 39% increase in the heat transfer coefficient using nanofluid containing 2% (v/v) copper nanoparticles. Wen and Ding (2005) witnessed a 40% enhancement in the heat transfer coefficient for a nanofluid containing 1.25% (v/v) alumina nanoparticles.

It is well known that the heat transfer fluids are poor conductors of heat, (the thermal conductivity of water is about 0.6 W m^{-1} k^{-1}) however the design of more conductive heterogeneous fluids provides an opportunity for improving thermal management efficiency in many applications. By adding a more thermally conductive liquid or solid the thermal conductivity of a base liquid can be increased. Since the solid thermal conductivity is three orders of magnitude greater than that of liquids Das et al., 2008 have focused on liquids containing solid nanoparticles. Das et al. (2008) reported that there is an increase in the thermal conductivity of the heterogeneous system over that of the base liquid. There are conflicting results with respect to the effect of particle size as studied by Xie et al. (2002) and Kim et al. (2007). Das et al. (2008) and Keblinski et al. (2002) proposed the enhancement in the thermal conductivity for different types of mechanisms which include Brownian motion of particles, the ordering of the liquid molecules at the solid interface, and the clustering of nanoparticle.

Buongiorno (2006) dealt with almost all aspects of convective transport in nanofluids. He also studied heat transfer enhancement and reported that there is no perfect reasoning for the abnormal increase of the thermal conductivity and viscosity on the convective transport of nanofluids. Buongiorno also explains that the dispersion of the suspended nanoparticles, does not contribute to convective heat transfer enhancement as this effect is too small for its contribution.

A sum of the base fluid velocity and a relative velocity (slip velocity) amounts to the absolute velocity of nanoparticles. This was noticed by Buongiorno (2006) and furthermore he took into account agencies namely; inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity settling and termed these as seven