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REVIEW

The response of nonlinear controlled system under an external excitation via time delay state feedback

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Abstract An analysis of primary, superharmonic of order five, and subharmonic of order one-three resonances for non-linear s.d.o.f. system with two distinct time-delays under an external excitation is investigated. The method of multiple scales is used to determine two first order ordinary differential equations which describe the modulation of the amplitudes and the phases. Steady-state solutions and their stabilities in each resonance are studied. Numerical results are obtained by using the Software of Mathematica, which presented in a group of figures. The effect of the feedback gains and time-delays on the non-linear response of the system is discussed and it is found that: an appropriate feedback can enhance the control performance. A suitable choice of the feedback gains and time-delays can enlarge the critical force amplitude, and reduce the peak amplitude of the response (or peak amplitude of the free oscillation term) for the case of primary resonance (superharmonic resonance). Furthermore, a proper feedback can eliminate saddle-node bifurcation, thereby eliminating jump and hysteresis phenomena taking place in the corresponding uncontrolled system. For subharmonic resonance, an adequate feedback can reduce the regions of subharmonic resonance response.

Contents

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1. Introduction

It is well known that changes in stability of response, undesirable bifurcations, high-amplitude vibrations, quasiperiodic motion, and chaotic behavior may occur and cause degradation or catastrophic failure of engineering structures, for example, buildings, bridges, and airplanes subjected to vibrations caused by wind, rotating engines, and cars, or other environmental disturbances. The task of suppressing the dangerous vibrations is very important for engineering science, and bifurcation control theory has received a great deal of attention in the last years and various papers have been dedicated to the control of resonantly forced systems in various engineering fields (Chen et al., 2000).

Time-delays, which are especially prevalent if a digital control system is being implemented, can limit the performance of the feedback controllers in practical mechanical or structural systems. In many cases, unavoidable time delays in controllers and actuators give rise to complicated dynamics and can produce instability of the controlled systems. On the other hand, the time-delays can deliberately be implemented to achieve better system behavior when control is applied with the time-delays (Aernouts et al., 2000). The effect of the feedback gains and time-delays on the dynamical behavior of the controlled system is thus required to be investigated for the design of optimal controllers. The non-linear system with the time-delays has been an active topic of research over the past decades (Atay, 1998; Plaut and Hsieh, 1987a,b; Moiola et al., 1996; Hy and Zh, 2000; Hu et al., 1998; Xu and Chung, 2003; Li et al., 2006).

Atay (1998) studied the effect of delayed position feedback on the response of a van der Pol oscillator. Plaut and Hsieh (1987a) numerically analyzed the steady response of a non-linear one-degree-of-freedom mechanism with the time-delays for various sets of parameters, by a Runge–Kutta numerical integration procedure. It was found that the response might be periodic, chaotic or unbounded. By the method of multiple scales, the same authors Plaut and Hsieh (1987b) studied the effect of a damping time-delay on non-linear structural vibrations and analyzed six resonance conditions. They gave the results in a number of figures for the steady state response amplitude versus the excitation frequency and amplitude. Moiola et al. (1996) considered Hopf bifurcations in nonlinear feedback systems with time delay by using the frequency-domain approach. Two simple examples of non-linear autonomous delayed systems were presented. The computation of the two periodic branches near a degenerate Hopf bifurcation point was given. Hy and Zh (2000) considered controlled mechanical systems with time delays and, in particular, primary resonance and subharmonic resonance of a harmonically forced Duffing oscillator with time delay (stabilization of periodic motion and applications to active chassis of ground vehicles were discussed). Hu et al. (1998) considered primary resonance and 1/3 sub-harmonic resonance of a forced Duffing oscillator with time-delay state feedback. Using the multiple scales method, they demonstrated that appropriate choices of the feedback gains and the time delay are possible for better vibration control. Xu and Chung (2003) discussed a Duffing–van der Pol oscillator with time-delayed position feedback and found two routes to chaos (period-doubling bifurcation and torus breaking). Li et al. (2006) considered the response of a Duffing–van der Pol oscillator under delayed feedback control and found that unwanted multiple solutions can be prevented. It is also shown that coupled nonlinear state feedback control can be replaced by uncoupled nonlinear state feedback control. Maccari (2008) investigated the periodic solutions for parametrically excited system under state feedback control with a time delay. Using the asymptotic perturbation method, two slow-flow equations for the amplitude and phase of the parametric resonance response are derived. It is demonstrated that, if the vibration control terms are added, stable periodic solutions with arbitrarily chosen amplitude and phase can be accomplished. Therefore, an effective vibration control is possible if appropriate time delay and feedback gains are chosen. In recent work, a new time-delayed feedback control method for nonlinear oscillators has been proposed. The method has been used to suppress high-amplitude response and two-period quasiperiodic motion of a parametrically or externally excited van der Pol oscillator (Maccari, 2001, 2005). In particular, it has been shown that vibration control and quasiperiodic motion suppression are possible for appropriate choices of time delay and feedback gains. The method has also been applied to the primary resonance of a cantilever beam (Maccari, 2003) and to direct and parametric excitation of a nonlinear cantilever beam of varying orientation (Yaman, 2009). El-Gohary and El-Ganaini (2012) considered how to control the dynamic system behavior represented by a beam at simultaneous primary and sub-harmonic resonance condition, where the system damage is probable. Control is conducted via time delay absorber to suppress chaotic vibrations. A comprehensive investigation of the effect of the time delay on the control of a beam when subjected to multi-parametric excitation forces is presented.

The main objective in this paper is to study the non-linear dynamical behavior of a harmonically excited non-linear single-degree-of-freedom (s.d.o.f.) system and its control by the appropriate choice of feedback gains and two distinct time-delays under primary, super-harmonic of order five and sub-harmonic resonance of order 1/3. Two time-delays are proposed in the proportional and derivative feedback. The governing equation of motion is assumed in the following form:

$$\ddot{x} + \omega_0^2 x + \varepsilon(2\mu\dot{x} + \alpha_1 x^2 + \alpha_2 x^3 + \alpha_3 x^4 + \alpha_4 x^5) = K\cos(\Omega t) + u(x, \dot{x}) \tag{1}$$

where,

$$u(x, \dot{x}) = 2\varepsilon[d_m x(t - \tau_1) + d_n \dot{x}(t - \tau_2)].$$