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REVIEW

A mathematical model for the distribution of heat through pavement layers in Makkah roads

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Abstract Pavement layers are the predominant type used in Saudi Arabian roads, it is necessary to have roads with excellent pavements from the structural and functional points of view. The heat distribution is highly influential in the pavement's structural design and has a large effect on indirect tensile stiffness modulus (ITSM). In this study, heat distribution through all the layers of the pavement will be studied. The mathematical technique used in the present analysis is the parameter-group transformation, the linear transformation group approach is developed to solve the heat diffusion problems in the presence of thermal conductivity and heat capacity. These problems obeyed an unusual power law relation, subject to nonlinear boundary conditions due to radiation exchange at the interface according to the fourth power law. The group theoretic approach shrinks the number of independent variables by one, therefore a nonlinear ordinary differential equation is obtained instead of the given nonlinear partial differential. The Runge–Kutta shooting method is used to solve the resulting nonlinear ordinary differential equation to determine heat distribution in the pavement layers of "Makkah" roads.

Contents

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1. Introduction

The road system in Saudi Arabia is one of the main transportation systems, which is developed in a rapid manner. Presently, the road construction program is still under way in all the regions of Saudi Arabia. The main function of these roads is to connect the cities, towns and villages as far as possible throughout the kingdom so; it is necessary to have roads with excellent pavements from the structural and functional points of view.

Pavement layers have been the predominant type used in Saudi Arabian roads and in other parts of the world, where the majority of paved surfaces fall under the overall category of pavement layers. Pavements may be classified as a conventional or a full depth pavement. Conventional pavements are layered systems that consist of an asphalt mixture over one or more granular layers (base and sub-base) which, together are constructed over sub-grade soil. Granular base and sub-base layers are essential components of a pavement system where their function is to reduce traffic induced stresses in the pavement structure and to minimize rutting in the base, sub-base and sub-grades.

In this study, heat distribution through all the layers of pavement in Makkah roads will be studied. The mathematical technique used in the present analysis is the parameter-group transformation. The foundation of the group-theoretic method is contained in the general theory of continuous transformation groups that were introduced and treated extensively by Lie (1881). The group methods, as a class of methods that lead to a reduction of the number of independent variables, were first introduced by Birkhoff (1948), where he made use of one-parameter transformation groups. Morgan (1952) presented a theory that has improvements over earlier similarity methods. In 1991, AbdelMalek et al. have applied group methods to study fluid flow and heat transfer characteristics for steady laminar free convection on a vertical circular cylinder. In 2005, Helal et al. have applied the group method analysis intensively, to study some problems in free-convective laminar boundary layer flow on a non-isothermal vertical circular cylinder. The advantages of the method are due to simplicity, and in reducing the number of independent variables by one, consequently, yielding complete results with lesser efforts. Hence it is applicable to solve a wider variety of nonlinear problems. Group-theoretic methods provide a powerful tool since they are not based on linear operators, superposition, or any other aspects of linear solution techniques. Therefore, these methods are applicable to nonlinear differential models, which will be used in this study.

2. Literature review

The pavement typically consists of a wearing course bituminous composite built over a base course and sub-base resting on a compacted sub-grade (Yoder and Witzak, 1975). The base may be either stabilized by asphalt, cement, lime, or other stabilizers; or untreated using granular material having specific physical properties.

The pavement should be designed to resist rutting, thermal and load induced cracking, moisture damage (stripping), and possess adequate fatigue life. Additional requirements include; durability, skid resistance and economy. The extent to which the asphaltic surface agrees with these requirements is mainly determined by the mixture constituents, mixture design, thickness design, construction factors and environmental conditions (Alawi, 2000).

Sub-grade characteristics and performance are influential in pavement structural design. Characteristics such as heat distribution, load bearing capacity, moisture content and expansiveness will influence not only structural design but also long-term performance and cost. Sub-grade materials are typically characterized by their resistance to deformation under load, which can be either a measure of their strength or stiffness. In general, the more resistant to deformation a sub-grade is, the more load it can support before reaching a critical deformation value. Three basic sub-grade stiffness/strength characterizations are commonly used in California Bearing Ratio (CBR).

In the years 2004–2008 the maximum air temperatures at the city of Makkah in Saudi Arabia were found to be 48 °C, 47 °C, 48 °C, 47 °C and 48 °C, respectively, while the minimum temperatures for the city were found to be 19 °C, 18 °C, 20 °C, 19 °C and 19 °C, respectively (Surface Annual Climatological Report, 2008).

Pavement temperatures remain in the range of 25–50 °C for most of the year in the eastern regions of Saudi Arabia. High temperatures (>50 °C) exist for about 26% of the year in the upper layer, while very high temperatures (>65 °C) are attained at a depth of 20 mm for only 2% of the time. Low temperatures (<10 °C) were observed at depths of 20 mm for only 3% of the time. These results were obtained from annual temperature distribution data (Al-Abdulwahhab and Ramadhan, 1995).

3. Mathematical formulation of the problem

The energy equation in a rectangular coordinate system can be used to determine heat distribution in pavement layers; wearing course and base course (which are asphaltic concrete mixtures), sub-base and sub-grade (unbound materials). Fig. 1 represents an example of a pavement consisting of wearing course (5 cm thick), base course (8 cm thick), and a sub-base of 25 cm thick and sub-grade of 300 cm, respectively. A heat transfer from the surface of the region takes place simultaneously by convection to an external environment having a heat transfer coefficient that varies with respect to time "$h(t)$" and by radiation to the surrounding, the temperature of this surrounding is maintained at "$q(t)$". Finally, the initial temperature is assumed to be zero throughout the solid.

The above transient heat conduction problem can be described by the following governing equation in dimensionless form (Davies, 1985, 1988) as:

$$\rho_i(x)C_i(T_i)\frac{\partial T_i}{\partial t} = \frac{\partial}{\partial x}\left(K_i(T_i)\frac{\partial T_i}{\partial x}\right),$$