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ORIGINAL ARTICLE

Mixed convective flow of immiscible fluids in a vertical corrugated channel with traveling thermal waves

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Abstract Fully developed laminar mixed convection in a corrugated vertical channel filled with two immiscible viscous fluids has been investigated. By using a perturbation technique, the coupled nonlinear equations governing the flow and heat transfer are solved. The fluids are assumed to have different viscosities and thermal conductivities. Separate solutions are matched at the interface using suitable matching conditions. The velocity, the temperature, the Nusselt number and the shear stress are analyzed for variations of the governing parameters such as Grashof number, viscosity ratio, width ratio, conductivity ratio, frequency parameter, traveling thermal temperature and are shown graphically. It is found that the Grashof number, viscosity ratio, width ratio and conductivity ratio enhance the velocity parallel to the flow direction and reduce the velocity perpendicular to the flow direction.

1. Introduction

Mixed convection is defined as a heat transfer situation where both natural convection and forced convection heat transfer mechanisms interact. In the past thirty years, mixed convection in a vertical heated channel has received considerable attention due to its extensive practical applications, including turbine rotor blade internal cooling systems, cooling of nuclear reactors and electronic components. From a technological point of view, the study of viscous fluids bounded by corrugated surfaces is of special interest and has practical applications in the cooling of electronic devices and systems, enhancing the heat transfer efficiency of industrial transport processes. The problem of viscous flow in a wavy channel was first treated analytically by Burns and Parks (1967). Later on Goldstein and Sparrow (1977), O'Brien and Sparrow (1982), Vajravelu (1989) and Saniei and Dini (1993) studied the flow through a corrugated channel.

Wang and Vanka (1995) determined the rates of heat transfer for flow through a periodic array of wavy passages. Malashetty et al. (2001a) studied the magnetoconvective flow and heat transfer between a vertical wavy wall and a parallel flat wall. Wang and Chen (2001) analyzed the rate of heat transfer for flow through a sinusoidal curved channel. A numerical study of mixed convection heat and mass transfer along a vertical wavy surface has been carried out by Jang and Yan (2004). Yao (2006) used finite difference methods to analyze the problem of natural convection boundary layer flow along a complex vertical surface represented by two sinusoidal functions. He found that the total heat-transfer rates for a complex surface are greater than those for a flat surface. Kuhn

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Nomenclature

a	amplitude
$C_p^{(j)}$	specific heat at constant pressure
C_p	ratio of specific heat at constant pressure
g	acceleration due to gravity
Gr	Grashof number $(h^{(1)^3} g\beta^{(1)}\Delta T/v^{(1)^2})$
h	width ratio $(h^{(2)}/h^{(1)})$
$K^{(j)}$	thermal conductivity
k	thermal conductivity ratio $(K^{(2)}/K^{(1)})$
m	viscosity ratio $(\mu^{(1)}/\mu^{(2)})$
Nu	Nusselt number
$P^{(j)}$	pressure
p_s	static pressure
$p^{(j)}$	dimensionless pressure
Pr	Prandtl number $\left(C_p^{(1)}\mu^{(1)}/K^{(1)}\right)$
r	density ratio $(\rho^{(2)}/\rho^{(1)})$
$T^{(j)}$	temperature
$T^{*(j)},\phi^{(j)}$	dimensionless temperature
T_s	static temperature
t	time
$U^{(j)},V^{(j)}$	velocities along X and Y directions
$u^{(j)},v^{(j)}$	dimensionless velocities
$X^{(j)},Y^{(j)}$	space co-ordinates
$x^{(j)},y^{(j)}$	dimensionless space co-ordinates

Greek symbols

$\beta^{(j)}$	coefficient of thermal expansion
β	ratio of coefficient of thermal expansion $(\beta^{(2)}/\beta^{(1)})$
ε	dimensionless amplitude parameter $(a/h^{(1)})$
λ	wave length
$\lambda^{(j)}$	dimensionless wave number $(\lambda^{(j)}/h^{(j)})$
$\mu^{(j)}$	viscosity
$v^{(j)}$	kinematic viscosity $(\mu^{(j)}/\rho^{(j)})$
θ	traveling thermal temperature
$\rho^{(j)}$	density
ρ_0	static density
τ	skin friction
ω	frequency parameter
ψ	stream function

Superscript

j = 1,2	where 1 and 2 refer quantities for the fluids in region-I and region-II respectively.

Subscript

0	mean part
1	perturbed part

and Rohr (2008) experimentally investigated mixed convective flow over a wavy wall.

One geometry of the flow passage that is very simple and may be used to enhance the exchanger performance is that formed by wavy walls. Wavy channels are easy to fabricate and can provide significant heat transfer enhancement if operated in an appropriate (transitional) Reynolds-number range. Therefore, wavy passages have been considered in several earlier studies as a means to enhance heat/mass transfer in compact exchange devices. Both corrugated and converging-diverging cross-sections have been studied experimentally and numerically. An important observation made is that wavy passages do not provide any significant heat transfer enhancement when the flow is steady. However, if the flow is made unsteady (either through external forcing or through natural transitioning to an unsteady state) significant increases in heat exchange are observed.

In realistic situations, however, the fluid system often consists of two (and possibly more) separate, immiscible liquids, a layer of one liquid overlying a layer of another liquid. The problem formulation now contains additional dynamical ingredients such as the interfacial stresses and the deformation of the interface shape. Also, a multi-layered liquid arrangement provides an improved model for the buoyancy-driven convection process in growing high-quality crystals.

The application of the two-fluid model is dependent on the presumed interface shape (either plane or curved) and on the availability of reliable closure relations for the wall shear and interfacial shear stresses (averaged over the corresponding wetted perimeter) in terms of the local/instantaneous holdup and velocities. These closure relations should correctly represent the effects of the system's parameters (e.g. fluids' flow rates and physical properties).

Meyer and Garder (1954) were the first authors to publish a paper on the mechanics of two immiscible fluids in porous media. Loharsabi and Sahai (1998) analyzed the flow of two immiscible fluids in a parallel plate channel assuming the continuity of velocity and thermal equilibrium at the interface. Several researchers have assumed that separated two-phase flow can be well represented by the superimposition of two single-phase flows separated by a flat interface. The first exact solution for the fluid flow in the interface region was presented in Vafai and Kim (1990). In that study, the shear stress in the fluid and the porous medium were taken to be equal at the interface region. Using this assumption, Malashetty and Leela (1992), Malashetty et al. (2001b, 2004), Umavathi et al. (2005, 2007, 2008a,b) and Prathap Kumar et al. (2011a,b) studied the flow and heat transfer of different immiscible fluids through channels. Most recently Umavathi and Shekar (2011, 2012) studied the mixed convection flow of immiscible fluids in a vertical corrugated channel.

In the literature, numerous experimental and theoretical studies have been reported concerning the heat transfer in the corrugated surface for the one-fluid model. Keeping in view the various applications of the two-fluid model, we were motivated to analyze the flow nature of two immiscible fluids in a vertical corrugated channel for unsteady flow. The temperature and velocity distributions are simulated by the perturbation method.