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ORIGINAL ARTICLE

# Design of optimal Mamdani-type fuzzy controller for nonholonomic wheeled mobile robots

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Nonholonomic wheeled mobile robot (WMR);
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**Abstract** In this paper, in order to cope with both parametric and nonparametric uncertainties in the robot model, an optimal Mamdani-type fuzzy logic controller is introduced for trajectory tracking of wheeled mobile robots (WMRs). The dynamic model of a nonholonomic mobile robot was implemented in the Matlab/Simulink environment. The parameters of input and output membership functions, and PID controller coefficients are optimized simultaneously by random inertia weight Particle Swarm Optimization (RNW-PSO). Simulation results show the system performance is desirable.


## 1. Introduction

The problem of motion control of wheeled mobile robots (WMRs) has been widely investigated in recent years. A significant motion control problem is trajectory tracking which is associated with the design of a controller to force a WMR to track a trajectory. Different types of control algorithms for trajectory tracking problem are proposed in the literatures (Dixon and Dawson, 2000; Fukao et al., 2000; Masahiro Oya, 2003; Wenjie Dong and Kuhnert, 2005).

Lately, intelligent methods such as neural networks (NNs) and fuzzy logic have been applied in controller designs to cope with different uncertainty problems in the system (Tang et al., 2006; Fierro and Lewis, 1998). In recent years, fuzzy logic control methods have been applied by many researchers to overcome disturbances and dynamic uncertainties of mobile robots. In Das and Kar (2006) a control method which enables the integration of a kinematic controller and an adaptive fuzzy controller for trajectory tracking is developed for nonholonomic mobile robots. In Abdessemed et al. (2004) a genetic algorithm to extracting the rules of a fuzzy controller intended to control the end effector motion of a planar manipulator in purpose to follow a prescribed trajectory is proposed. In Xianhua et al. (2005) a path tracking scheme for a mobile robot based on fuzzy logic and predictive control is presented, where the predictive control is used to predict the position and the orientation of the robot, while the fuzzy control is used to deal with the non linear characteristics of the system.

Fuzzy logic control has been known for its efficacy in the control of wheeled mobile robots in order to perform missions

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in uncertain conditions where robustness properties must be intended in the control procedure (Chiu et al., 2005; Das and Kar, 2006; Khooban et al., 2012, 2013; Khooban and Soltanpour, 2013; Khooban et al., 0000). The significant drawback of fuzzy controllers is the lack of any systematic methods to define fuzzy rules and fuzzy membership functions. Most fuzzy rules are based on personal information and differ for different persons despite the same system performance. On the other hand, it is difficult to expect that any given expert's knowledge gathered in the form of the fuzzy controller leads to an optimal solution. Consequently, some effective approaches for tuning the membership function and control rules without a trial and error method are significantly required.

Recently, PSO algorithm has become available and promising techniques have been developed for real world optimization problems (Bergh and Engelbrecht, 2006). Compared to GA, PSO needs less time for each functional evaluation as it does not use many of the GA operators like mutation, crossover and selection operator (Kennedy and Eberhart, 1999). Due to its simple concept, easy implementation and fast convergence, nowadays PSO has gained much attention and wide applications in different fields.

Motivated by the aforementioned researches, the goal of this paper is to design an optimal Mamdani-type fuzzy controller for wheeled mobile robots (WMRs) to track a mobile object. To achieve this objective, the random inertia weight Particle Swarm Optimization (RNW-PSO), which is an improved algorithm of PSO, is employed to choose the best parameters i.e.amount of the input and output membership functions and also the closed-loop weighting factors.

## 2. Dynamic model of WMR

The configuration of nonholonomic WMR is shown in Fig. 1. Using the Euler–Lagrange formulation, the dynamic model of WMRs can be explained by Fierro and Lewis (1995), Fukao et al. (2000):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - A^T(q)\lambda \quad (1)$$

where $M(q) \in \Re^{n\times n}$ is the symmetric and positive definite inertia matrix, $C(q,\dot{q}) \in \Re^{n\times n}$ is the centripetal matrix,

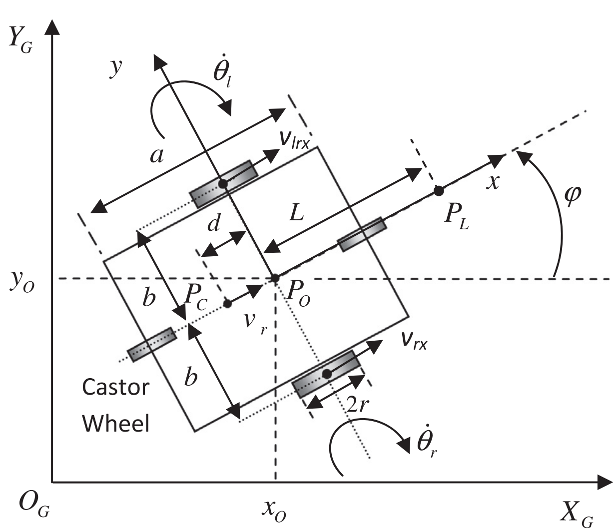


**Figure 1** Configuration of nonholonomic WMR.

$F(\dot{q}) \in \Re^{n\times 1}$ is the vector of surface friction, $G(q) \in \Re^{n\times r}$ is the gravitational vector, $\tau_d$ indicates the limited unknown disturbances including unstructured dynamics, $B(q) \in \Re^{n\times r}$ is the input transformation matrix, $\tau \in \Re^{r\times 1}$ is the input vector, $A(q) \in \Re^{m\times n}$ is the matrix related to the restrictions, and $\lambda \in \Re^{n\times 1}$ is the vector of restriction forces. Surface friction is intended as:

$$f(\dot{q}) = F_v\dot{q}_i + F_d\,\mathrm{sgn}(\dot{q}_i) \quad (2)$$

where $F_v$ and $F_d$ are the coefficients of the viscous and dynamic frictions, respectively. The dynamics of the DC servomotors, which drive the wheels of the robot, is stated as follows

$$\begin{aligned} &\tau_s = K_T i_a \\ &Li_a + Ria + K_e + \dot{\Phi}_e = u \end{aligned} \quad (3)$$

where $\boldsymbol{\tau}_e \in R^n$ is the vector of torque produced by the motor, $\mathbf{K}_T \in R^{m\times n}$ is the positive definite constant diagonal matrix of the motor torque constant, $\mathbf{i}_a \in R^n$ is the vector of armature currents; $L$, $R$, and $K_e$ are the diagonal matrix of armature inductance, armature resistance and back electromotive force constant of the motors, respectively; and $\Phi_e$ is the angular velocity of the actuator motors. The motor torque $\tau_s$ and the wheel torque $\tau$ are associated with the gear ratio $N$ as

$$\tau = N\tau_s \quad (4)$$

where $N$ is a positive definite and constant diagonal matrix. The angular velocities of the actuators $\dot{\Phi}_e$ is associated with the wheel angular velocities $v_w$ as

$$Vw = N^{-1}\dot{\Phi}_e \quad (5)$$

without considering the armature inductance and due to the Eqs. (4) and (5), Eq. (3) can be written as follows

$$\tau = K_1 u - K_2 v_w \quad (6)$$

where $K_1 = (NK_T/R_a)$ and $K_2 = NK_eK_1$. The relationship between the wheel angular velocities $v_w$ and the velocity vector $v$ is

$$v_m = \begin{pmatrix} w_r \\ w_l \end{pmatrix} = \begin{pmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & \frac{-b}{r} \end{pmatrix} \quad (7)$$

Substituting Eqs. (6) and (7) in Eq. (1), the equation of WMR, which includes actuator dynamics, can be achieved as follows

$$\begin{aligned} &M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d \\ &\quad = B(q)\left(K_1 u - K_2 \sum v\right) - A^T(q)\lambda \end{aligned} \quad (8)$$

The kinematic model of WMR can be written as follows

$$\dot{q} = S(q)v \quad (9)$$

By taking time derivative of the kinematic model (8), the robot dynamics (8) can be converted as follows

$$\overline{M}\dot{v} + \overline{C}v + \overline{F} + \overline{\tau_d} = K_1\overline{B}u \quad (10)$$

where

$$\overline{M} = S^T MS, \overline{C} = S^T M\dot{S} + S^T CS + K_2\overline{B}\Sigma \quad (11)$$

and $\overline{B} = \mathrm{S^T}B$, $\overline{F} = \mathrm{S^T}F$, $\bar{\tau}_d = \mathrm{S^T}\tau_d$

$\overline{B} = \mathbf{S}^T B, \overline{F}, \bar{\tau}_d = S^T\tau_d$

Based on Eq. (11), the input voltages of the wheel actuators are considered as the control inputs.