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ORIGINAL ARTICLE

Two stage group acceptance sampling plan for half normal percentiles

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Abstract In this paper, a time truncated life test based on two stage group acceptance sampling plan for the percentile lifetime following half-normal distribution is proposed. The optimal parameters for the proposed plan are determined such that both producer's and consumer's risks are satisfied simultaneously for the given experimentation time and sample size. The efficiency of the proposed sampling plan is discussed in terms of average sample number with the existing sampling plan. The proposed sampling plan is explained with the help of industrial examples.

1. Introduction

In today's modern industrial environment it has become mandatory to produce high quality products with the help of modern statistical quality control (SQC) techniques. Quality of any product is a very important factor which leads to business success, growth and an improved competitive position (see Balakrishnan et al. (2007)). The SQC techniques are very essential now days for any manufacturing process which will help to improve the quality of the product, since the application of SQC tools will reduce the variability of the process and products. SQC plays a substantial role for the success of any industry. SQC involves a set of operating activities that an enterprise implements in order to get certified that the quality of its products is at required levels of the consumers. According to Montgomery (2009), one can evaluate the quality of the product in terms of its durability, serviceability, performance, aesthetics, features, reliability, quality and its conformance to standards collectively and they are termed as the dimensions of quality. Hence, we can say that in selecting among competing products and services, the quality has become the most significant factor for consumer's satisfaction.

The quality of any finished product can be judged by inspecting a few items taken randomly from a lot of products

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and the process of taking such samples is called the sampling. In quality management, the acceptance sampling plans are vital tools in making a decision about the product whether to accept or reject based on the inspection of sampled items and the sampling plans prescribe the experimenter how many items in the sample should be selected from the submitted lot for inspection and how many defectives can be allowed in this sample in order to satisfy both the producer and the consumer. The probability of rejecting a good lot is called the producer's risk and the chance of accepting bad lot is called the consumer's risk. The cost of any life test experiment is directly proportional to the sample size. Therefore, sampling plans that provide smaller sample size for inspection and minimize two risks are considered as efficient sampling plans.

In general, the acceptance sampling plans can be classified into two types namely attribute sampling plans and variables sampling plans. The attribute sampling plans are implemented for quality characteristics which are expressed on a "go, no-go" basis where as the variables sampling plans can be applied where the quality characteristics of interest are measured on a numerical scale (see Montgomery (2009)). Various types of sampling plans such as single sampling plan (SSP), double sampling plan (DSP), multiple sampling plan, and sequential sampling plan are available in the literature (see Schilling (1982)).

Group acceptance sampling plan (GASP) is one of the types of sampling plans which involves a number of testers to be used for testing so that cost and time can be reduced. The inspection of multiple items simultaneously can be made easy to the experimenter for testing. Two stage group acceptance sampling plan is the extension of GASP which involves two groups. The GASP is more advantageous than the conventional sampling plans in terms of minimum inspection so that the considerable testing time and cost can be reduced (see Aslam et al. (2009)). The advantage of two stage group sampling plan is that it reduces the average sample number (ASN) as compared to the GASP.

Several authors have investigated the sampling plans under various life time distributions, which are available in the literature of acceptance sampling (see for example, Kantam et al. (2001), Baklizi and El Masri (2004), Balamurali and Jun (2006), Tsai and Wu (2006), Jun et al. (2006), Aslam and Jun (2009), Aslam et al. (2009, 2011, 2013), Rao (2009), Lio et al. (2010), Rao and Kantam (2010)). Lu et al. (2013) proposed the SSP based on half normal distribution.

By exploring the literature on two stage group sampling plan, it can be seen that no work is available based on the half normal distribution. In this paper, we will present the designing of two stage group sampling plan when the life time of an item follows the half normal distribution. The structure of proposed plan is presented and efficiency is compared with the existing sampling plan. The application of the proposed sampling plan is explained with the help of industrial illustrative examples.

2. Half-normal distribution

As far as the variables sampling plans are concerned, the normal distribution is the most preferred statistical distribution. But for life testing problems, normal distribution is not preferred because of its range $(-\infty,\ \infty)$. However, one of the normal family distributions called the half-normal distribution is the widely used probability distribution for nonnegative data modeling, particularly, in life time testing. Luis et al. (2012) investigated the properties of half normal distribution. Khan and Islam (2012) investigated the maintenance performance of the system under half-normal failure lifetime model as well as a repair-time model. The probability density function (pdf) of a half normal distribution with 0 mean and its parameter θ with domain $x \in [0, \infty]$ is given by

$$f(x) = \frac{2\theta}{\pi} e^{-\frac{x^2\theta^2}{\pi}}, \quad x > 0,\ \theta > 0 \tag{1}$$

Its cumulative distribution function (cdf) is given by

$$F(x) = erf\left(\frac{\theta x}{\sqrt{\pi}}\right), \quad x > 0,\ \theta > 0 \tag{2}$$

Here *erf* is the "Error Function" defined by

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{3}$$

Consider that life time of product follows a half-normal distribution with σ as a scale parameter. Its cdf is given by

$$F(x) = erf\left(\frac{\theta\left(\frac{t}{\sigma}\right)}{\sqrt{\pi}}\right), \quad t > 0,\ \theta > 0,\ \sigma > 0 \tag{4}$$

The 100th percentile of the half normal distribution with $0 < q < 1$, is defined as

$$t_q = \frac{\sigma\sqrt{\pi}}{\theta} erf^{-1}(q), \quad \text{for } 0 < q < 1 \tag{5}$$

Here erf^{-1} is inverse function of error function. The Maclaurin series of $erf^{-1}(.)$ is given by

$$erf^{-1}(x) = \sqrt{\pi}\left(\frac{1}{2}x + \frac{1}{24}\pi x^3 + \frac{7}{960}\pi^2 x^5 + \frac{127}{80640}\pi^3 x^7 + \ldots\right) \tag{6}$$

According to Pewsey (2004), "If Z is a standard normal random variable, $Z \sim N(0,1)$, then $X = |Z|$ follows a standard positive half normal distribution and $-X = -|Z|$. follows a standard negative half-normal distribution. The half-normal distribution is a central chi-square distribution with one degree of freedom and a special case of truncated and folded normal distributions". The half-normal distribution is also a limiting case of skewed normal distribution (see Ayman and Kristen (2013)). The applications of half-normal in reliability analysis can be seen in Ayman and Kristen (2013). As half-normal distribution has positively skewed shape, there is a need to model monotone hazard rates. The half-normal distribution is very widespread model to describe the lifetime process of any device under fatigue (see Rodrigo et al. (2010)).

3. Two stage group acceptance sampling plan

Aslam et al. (2012) proposed the two stage group sampling plan. The operating procedure of this plan is explained below.

3.1. Stage one

i. Extract the first random sample of size n_1 from a lot submitted for inspection.