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ORIGINAL ARTICLE

Fully developed MHD natural convection flow in a vertical annular microchannel: An exact solution

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Abstract An exact solution of steady fully developed natural convection flow of viscous, incompressible, electrically conducting fluid in a vertical annular micro-channel with the effect of transverse magnetic field in the presence of velocity slip and temperature jump at the annular micro-channel surfaces is obtained. Exact solution is expressed in terms of modified Bessel function of the first and second kind. The solution obtained is graphically represented and the effects of radius ratio (η), Hartmann number (M), rarefaction parameter ($\beta_v Kn$), and fluid–wall interaction parameter (F) on the flow are investigated. During the course of numerical computations, it is found that an increase in Hartmann number leads to a decrease in the fluid velocity, volume flow rate and skin friction. Furthermore, it is found that an increase in curvature radius ratio leads to an increase in the volume flow rate.

1. Introduction

Recently, a growing interest in micro-channel fluid mechanics and heat transfer has emerged because of possible cooling applications in space systems, manufacturing and material processing operations, and in high-power-density chips in supercomputers and other electronics (Al-Nimr and Khadrawi, 2004) . As this area continues to grow, it becomes increasingly important to understand the mechanisms and fundamental differences involved with fluid mechanics and heat transfer mechanisms in macro-channel and micro-channel.

A series of investigations have been conducted recently in the field of micro geometry flow. However, to cite a few works in this direction, Chen and Weng (2005) analytically studied the fully developed natural convection in open-ended vertical parallel plate micro-channel with asymmetric wall temperature distribution in which the effect of rarefaction and fluid wall interaction was shown to increase the volume flow rate and decrease the heat transfer. This result is further extended by taking into account suction/injection on the micro-channel walls by Jha et al. (2014). They concluded that skin frictions as well as the rate of heat transfer are strongly dependent on suction/injection parameter. In another work, Weng and

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Nomenclature

B_0	constant magnetic flux density
$C_{\rho 0}$	specific heat at constant pressure
F	fluid–wall interaction parameter, β_t/β_v
g	gravitational acceleration
k_1	radius of the inner cylinder
k_2	radius of the outer cylinder
Kn	Knudsen number, λ/w
M	Hartmann number
q	volume flow rate
Q	dimensionless volume flow rate
Pr	Prandtl number
r	dimensional radial coordinate
R	dimensionless radial coordinate
$\widehat{R}$	specific gas constant
T	temperature of fluid
T_0	reference temperature
T_1	temperature at outer surface of the inner cylinder
u	axial velocity
U	dimensionless axial velocity
w	dimensional gap between the cylinders
σ_t, σ_v	thermal and tangential momentum accommodation coefficients, respectively

Greek letters

α	thermal diffusivity
β_0	coefficient of thermal expansion
β_t, β_v	dimensionless variables
γ	ratio of specific heats
μ_0	dynamic viscosity
θ	dimensionless temperature
ρ_0	density
ν	fluid kinematic viscosity (μ_0/ρ_0)
η	ratio of radii (k_1/k_2)
λ	molecular mean free path
k_0	thermal conductivity
σ	electrical conductivity of the fluid
τ	skin-friction

Chen (2009) studied the impact of wall surface curvature on steady fully developed natural convection flow in an open-ended vertical micro-annulus with an asymmetric heating of the annulus surface. Recently, Jha et al. (in press) further extended the work of Weng and Chen (2009) by taking into account suction/injection on a vertical annular micro-channel. They discovered that skin-friction decreases at the outer surface of the inner porous cylinder with an increase of fluid–wall interaction parameter in the case of injection at the outer surface of the inner porous cylinder and simultaneous suction at inner surface of the outer porous cylinder while the result is just reverse at the inner surface of outer porous cylinder. In a related article, Avci and Aydin (2009) studied the fully developed mixed convective heat transfer of a Newtonian fluid in a vertical micro-annulus between two concentric micro-tubes. Recently, Jha and Aina (2014) further extended the work of Avci and Aydin (2009) to the case when the vertical micro-annulus formed by two concentric micro-tubes is porous, i.e. where there is suction or injection through the annulus surfaces.

On the other hand, the MHD phenomenon has received considerable attention during the last two decades due to its importance from the energy generation point of view, and one may envisage MHD generators for power generation. MHD pumps are already in use in chemical energy technology for pumping electrically conducting fluids at some of the atomic energy centres. Besides these applications, when the fluid is electrically conducting, the free convection flow is appreciably influenced by an imposed magnetic field. Therefore, to refer to few works in this direction, Sheikholeslamia et al. (2014a) investigated the magnetic field effect on nanofluid flow and heat transfer in a semi-annulus enclosure via control volume based finite element method. Khan and Ellahi (2008) observed the effects of magnetic field and porous medium on some unidirectional flows of a second grade fluid. Farhad et al. (2012) examines the slip effect on hydromagnetic rotating flow of viscous fluid through a porous space. In another work, Ali et al. (2012) investigated the effects of slip condition on the unsteady magnetohydrodynamics (MHD) flow of incompressible viscoelastic fluids in a porous channel under the influence of transverse magnetic field and Hall current with heat and mass transfer. An analysis to investigate the combined effects of heat and mass transfer on free convection unsteady magnetohydrodynamics (MHD) flow of the viscous fluid embedded in a porous medium is presented by Ali et al. (2013a)

Some recent works related to the present investigation are found in the literature (Ali et al., 2013b, 2014; Sheikholeslamia et al., 2014b; Sheikholeslami et al., 2012a,b; Ashorynejad et al., 2013). In Ali et al. (2013b), Farhad et al. presented an exact analysis of combined effects of radiation and chemical reaction on the magnetohydrodynamics (MHD) free convection flow of an electrically conducting incompressible viscous fluid over an inclined plate embedded in a porous medium. Ali et al. (2014) studied the unsteady free convection flow of a second grade fluid past an isothermal vertical plate oscillating in its plane with constant viscosity. Also, Sheikholeslamia et al. (2014b) studied the magnetohydrodynamic effect on natural convection heat transfer of Cu–water nanofluid in an enclosure with a hot elliptic cylinder. Sheikholeslami et al. (2012a) numerically examined the natural convection of nanofluids in a concentric annulus between a cold outer square cylinder and a heated inner circular. Flow and heat transfer of a nanofluid over a stretching cylinder in the presence of magnetic field has been investigated by Ashorynejad et al. (2013). Recently, Sheikholeslami et al. (2012b) presented the numerical solution for natural convection of nanofluids in a cold outer circular enclosure containing a hot inner sinusoidal cylinder. Sheikholeslamia et al. (2013) carried out a numerical investigation on natural convection nanofluid flow in a half annulus enclosure with one wall under constant heat flux in the presence of a magnetic field.

However, to the best knowledge of the authors, no studies have been carried out on the fully developed MHD natural