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ORIGINAL ARTICLE

Methodological insights for industrial quality control management: The impact of various estimators of the standard deviation on the process capability index

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Abstract Statistical quality control (SQC) is used by companies and industries for many reasons. For example, the process capability of machines is an important aspect of SQC, which consists in evaluating the ability of a production process to perform with the required specifications. In other words, the process capability measures the ability of a process of producing acceptable products according to the established specifications. The most common indicator used to measure the process capability is the process capability index, which depends on the process standard deviation. In practice, the standard deviation is unknown, and the process capability index is thus estimated by using an estimator of the process standard deviation. In this paper, we describe the most common estimators of the process standard deviation, and define the corresponding estimators of the process capability index. A bound for the bias ratio of the various estimators is obtained. Monte Carlo simulation studies are carried out to analyze the empirical performance of the various estimators of the process capability index. Empirical results indicate that biases can be obtained, specially in the presence of small samples. We also observe that the estimators of the process capability index based on sample ranges are less accurate than the alternative estimators.

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1. Introduction

The problem of ensuring the quality of products is a very common practice in many companies and industries. This issue is a clear example on the management literature regarding how managers take decisions based on data (see also Lynch, 2008; Parry et al., 2014). The set of statistical tools used to

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control and improve the quality of products is known as statistical quality control (SQC), and which involves various aspects. For example, control charts are used to monitor the quality of a process and determinate if this process is in a state of statistical control (in control), which would indicate that the production has a normal variation. An additional statistical tool within SQC is acceptance sampling, which consists in inspecting lots of products with the purpose of deciding whether they are accepted or not according to the results derived from the inspection. SQC also involves the capability analysis, which is the topic discussed in this paper. The capability analysis indicates if the process has the ability of producing acceptable products. An introduction to SQC can be seen in Montgomery (2009).

The process capability index is the main indicator used to measure the capability analysis. The process capability index evaluates a production process and indicates if the process is capable, i.e., it is prepared to produce items with the required specifications. The capability analysis is considered as a very important aspect in many manufacturing industries, and for this reason several researchers have conducted studies related to capability indices. Relevant references are Anis (2008), Besseris (2014), Bissell (1990), Boyles (1991), Chan et al. (1988), Chen and Ding (2001), Chen et al. (2001), Chen et al. (2003), English and Taylor (1993), Kane (1986), Kotz and Jhonson (2002), Kotz and Lovelace (1998), Kushler and Hurley (1992), Luceo (1996), Pearn et al. (1992), Porter and Oakland (1991), Rodriguez (1992), Somerville and Montgomery (1996), Spiring et al. (2003), Yeh and Bhattcharya (1998), etc.

Note that the control charts and the capability analysis are related concepts. In particular, acceptable products are produced if the process is capable and in control before the production begins.

A process capability index is based on specification limits, also named as tolerances. We assume two-sided specification limits defined by the lower specification limit (LSL) and the upper specification limit (USL), and which generally indicate ranges of acceptance quality characteristics. In other words, a product is considered as acceptable if its characteristics are within the specification interval [LSL, USL]. For example, the specification limits for the volume of bottles may be specified as 2 liters ± 0.05 liters, which indicates that $LSL = 1.95$ liters and $USL = 2.05$ liters. One-sided specification limits can be also defined. For example, the volume of bottles may have the lower specification limit $LSL = 1.95$, but not an upper specification limit (see also Montgomery, 2009, p. 9).

A process capability index is also based on the process standard deviation, which is denoted as σ. In practice, the parameter σ is unknown, and the use of an estimator is required in this situation. Traditionally, the technique used for the estimation of σ consists in selecting m samples with the same size n. Simple random sampling without replacement is the most common sampling design used to select the various samples. Note that the m samples must be obtained when it is known that the process is stable. The information collected from these samples is used for the purpose of estimating σ. The most common estimators used to estimate the process standard deviation are based on the sample standard deviations and the sample ranges (see Chakraborti et al., 2008; Chen, 1997; Duncan, 1986; Jones et al., 2001; Luko, 1996; Luko, 1996; Chen, 1997, pp. 229 and 253; Ott, 1975; Vardeman, 1999; Wheeler, 1995; Woodall and Montgomery, 2000).

This paper discusses the estimation of the customary process capability index, which is defined as the ratio of the specification width $(USL - LSL)$ to the width of the process variability (6σ). Note that we consider 6σ for the width of the process variability because it is quite common in practice to use the criterium of 3σ control limits when dealing with control charts (see Chen, 1997; Montgomery, 2009, p. 184). The main objective of this paper is to analyze the empirical performance of various estimators of the process capability index and assuming different scenarios.

This paper is organized as follows. In Section 2 we describe the most common estimators of the process standard deviation σ. In Section 3 we define the customary process capability index, which in turn is used to define the various estimators of this index based on the estimators of σ described in Section 2. The main contribution of this paper can be found in Section 4, where we carry out various Monte Carlo simulation studies based on different scenarios. For example, we considered the classical example with data based on the Normal distribution, but we also considered non-normal data and off-center processes. The aim of this empirical study is to analyze the empirical performance of the various estimators of the process capability index in terms of relative bias and relative root mean square error. Empirical results indicate that the various estimators can be biased, specially for small sample sizes. We also observe that the estimators based on the sample ranges are less accurate than the alternative estimators. The use of the Gamma distribution does not have an important impact on the empirical performance of the various estimators. This conclusion is also observed when off-center processes are considered. Finally, the empirical results indicate that the use of the Uniform distribution has a relevant impact on estimators based on the sample ranges. Finally, in Section 5, the main conclusions derived from the various Monte Carlo simulation studies are presented.

2. The customary estimators of the process standard deviation

In this section, we describe the most common estimators of the process standard deviation used in practice.

Let σ be the true standard deviation of a production process. It is quite common to assume that σ is unknown, since it is unlikely to know this parameter in practice. In particular, most control charts are based on estimators of σ (see Chakraborti et al., 2008; Chen, 1997; Jones et al., 2001; Montgomery, 2009, p. 228). In this situation, the process capability index also requires the estimation of the true standard deviation σ.

The unknown parameters related to a process are generally estimated by using m samples, which must be selected when the process is believed to be in control. It is also quite common to assume that the various samples have the same size n. Note that expressions for the case of samples with different sizes can be easily derived from the existing literature (see, for example, Montgomery, 2009, p. 255). It is also common to use simple random sampling without replacement for the problem of selecting the m samples. Note that the problem of selecting the best sampling design for the selection of the various samples is also a topic which is beyond the scope of this paper.