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SHORT COMMUNICATION

Fluid flow and radiative nonlinear heat transfer over a stretching sheet

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Abstract In the present paper, we endeavor to perform a numerical analysis in connection with the boundary layer flow induced in a quiescent fluid by a continuous sheet stretching with velocity $u_w(x) \sim x^{1/3}$ with heat transfer. The effects of thermal radiation using the nonlinear Rosseland approximation are investigated. We search for similarity solutions and reduce the problem to a couple of ordinary differential equations containing three dimensionless parameters: the radiation parameter N_R, the temperature ratio parameter θ_w and the Prandtl number Pr. The computational results for velocity, temperature and heat transfer characteristics are presented in both graphical and tabular forms.

1. Introduction

In contrast to the well-known Blasius flow problem (see, for instance, Cortell, 2005), and ref. therein) which involves laminar viscous boundary layer fluid flow above a fixed flat plate, the flow of a viscoelastic fluid over a rigid plate moving steadily in an otherwise quiescent fluid is sometimes referred to as Sakiadis flow (see Sakiadis, 1961) after the pioneering work of that researcher. Ahmad and Al-Barakati (2009) obtained an approximate analytical solution of the Blasius problem. Sakiadis flow's studies were recently dealt by Sadeghy et al. (2005) in their work on boundary layer of an upper-convected Maxwell fluid flow where the role played by fluid's elasticity in the flow characteristics was analyzed.

When the difference between the sheet and the ambient temperature is large the thermal radiation effects become important, and also at high operating temperature the presence of thermal radiation alters the thermal boundary layer structure and the rate of heat transfer also results altered. In such industrial processes knowledge of radiative heat transfer becomes relevant. Abo-Eldahab and Azzam Gamal El-Din (2005) gave examples like nuclear power plants, gas turbines, satellites, etc. Viskanta and Grosh (1962) studied boundary layer flow in thermal radiation absorbing and emitting media by using the Rosseland approximation (Rosseland, 1931). Numerical results for hydro-magnetic mixed convection flow over a permeable non-isothermal wedge were reported by Prasad et al. (2013). Hossain et al. (1999) studied thermal radiation's effects using the Rosseland diffusion approximation on natural convection flow of an optically thick viscous

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Nomenclature

c_P	Specific heat of the fluid at constant pressure J kg^{-1} K^{-1}
$\frac{d}{d\eta}$	Derivative with respect to η
f	Dimensionless stream function
k^*	Rosseland mean absorption coefficient m^{-1}
k	Fluid thermal conductivity W m^{-1} K^{-1}
L	Characteristic length m
N_R	Radiation parameter
Pr	Prandtl number
q_r	Component of radiative heat flux in y direction W m^{-2}
T	temperature K
u, v	Velocity components along x and y directions, respectively m s^{-1}
x, y	Cartesian coordinates along the plate and normal to it, respectively. m
Greek symbols	
α	Thermal diffusivity m^2 s^{-1}
η	Dimensionless similarity variable
θ	Dimensionless temperature
μ	Absolute viscosity kg m^{-1} s^{-1}
υ	Kinematic viscosity m^2 s^{-1}
ρ	Density kg m^{-3}
σ^*	Stefan-Boltzmann constant W m^{-2} K^{-4}
Subscripts	
w, ∞	Conditions at the surface and within the free stream, respectively
Superscript	
prime	Derivative with respect to η

incompressible flow past a uniformly heated vertical porous surface with constant suction, and further, Hossain et al. (2001) analyzed the effect of variable viscosity on this type of flow. On the other hand, Raptis and Perdikis (1998) used a linearized form of the aforesaid Rosseland approximation in view to analyze the steady flow of a visco-elastic fluid past an unmoving surface. These simplifications permit an easier analysis, and many investigations (see, for instance, Cortell, 2008a,2011a; Abdul Hakeem et al., 2013) have been carried out in the recent past that deal with thermal studies by applying the cited linearized form, which is derived by assuming sufficiently small temperature differences within the flow that may assure to express T^4 as a linear function of temperature. Studies about motion and mass transfer with chemically reactive species in a porous space were recently undertaken by Cortell (2007a, 2007b). Moreover, treatments to the radiative heating for flows generated by linear/nonlinear stretching sheets enclosing magneto-hydrodynamics, non-Newtonian fluids, porous media, etc. constitute analytical or numerical attempts which have been made in the recent past (Arpaci, 1968; Cortell 2008b, 2011b, 2012a, 2012b; Turkyilmazoglu, 2011; Misra and Sinha, 2013). The problem of steady micropolar fluid flow past a stretching surface has been devised by many authors (see, for instance, Ishak, 2010; Hsiao, 2010) and even, very recently, unsteady fluid flow with (Hsiao, 2012) or without (Bachok et al., 2011) thermal radiation effects has also been analyzed.

One of the objectives of the present paper is to extend the investigation of Cortell (2008c) to analyze the Sakiadis flow generated by a sheet stretched with a velocity which is assumed to be proportional to the $x^{1/3}$quantity, x being the distance from the slit. We also assume appropriate boundary conditions for the energy equation that may assure the existence of similarity solutions (i.e., constant temperature at the surface) when radiative nonlinear heat transfer is studied. Very recently, Rahman and Eltayeb (2013), Pantokratoras and Fang (2013) used the Rosseland diffusion approximation in studying radiative nonlinear heat transfer in different geometries. Unlike the linearized Rosseland approximation which is derived by assuming sufficiently small temperature differences between the plate and the ambient fluid, when use is made of the nonlinear Rosseland diffusion approximation one can obtain results for both small and large differences between T_w (constant surface temperature) and T_∞ (the constant ambient fluid temperature). It is also known that the inclusion of nonlinear radiative effects in the energy equation had led to a highly nonlinearity in the governing equations (see El-Hakim and Rashad, 2007).

The fluid is at rest and the motion is created by the surface whose velocity varies nonlinearly with the distance x from a fixed point and the sheet is held at a temperature higher than the temperature T_∞ of the ambient fluid. For the stated problem and to our knowledge, the presented data on thermal analysis have not been considered before.

This paper aims to find similarity numerical solutions for problem above-mentioned. In Section 2 we shall examine the analysis of the flow and its mechanical characteristics. Heat transfer of a viscous fluid over a nonlinear stretching sheet in the presence of thermal radiation will be analyzed in Sections 3–4 by means of the nonlinear Rosseland diffusion approximation. The paper ends with its conclusions in Section 5.

2. Flow analysis

Let us consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The fluid is assumed to be a gray, absorbing-emitting but non-scattering medium. Use is made of usual notation and then we can express the basic equations describing the conservation of mass and momentum in the boundary layer as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

where (x, y) denotes the Cartesian coordinates along the sheet and normal to it, u and v are the velocity components of the fluid in the x and y directions, respectively, and $\upsilon(=\frac{\mu}{\rho})$ is the