



ORIGINAL ARTICLE

Exp and modified Exp function methods for nonlinear Drinfeld–Sokolov system

J. Biazar ^{*}, Z. Ayati

Department of Mathematics, Islamic Azad University (Rasht Branch), P.O. Box 41335-3516, Rasht, Iran

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Abstract In this paper, Exp-function and its modification methods have been applied to obtain an exact solution of the nonlinear Drinfeld–Sokolov system (DS). Modification of the method was first introduced by the same authors. The prominent merit of this method is to facilitate the process of solving systems of partial differential equations. These methods are straightforward and concise by themselves; moreover, their applications are promising to obtain exact solutions of various partial differential equations. It is shown that the methods, with the help of symbolic computation, provide very effective and powerful mathematical tools for solving such systems.

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1. Introduction

Mathematical modelings of many real phenomena lead to a non-linear ordinary or partial differential equations in various fields of physics and engineering. There are some methods to obtain approximate or exact solutions of these kinds of equations, such as the tanh method (Wazwaz, 2005; Malfliet and Hereman, 1996), sine–cosine method (Wazwaz, 2006), homotopy perturbation method (Biazar and Ghazvini, 2007; He, 2005), variational iteration method (He, 1999; He, 2000),

Adomian decomposition method (Biazar et al., 2003), and many others (Wang, 1996; Abdou, 2007; Wang and Zhang, 2005; Wang et al., 2008). Most recently, a novel approach called the Exp-function method (He and Wu, 2006; Zhang, 2007; Biazar and Ayati, 2008) has been developed to obtain solutions of various nonlinear equations. The solution procedure of this method, by the help of any mathematical packages, say Matlab or Maple, is of utter simplicity. The modified version of this method was first presented in Biazar and Ayati (2009) by current authors. There, it was used to solve the system of partial differential equation directly and without change to ordinary differential equation.

In this paper, the nonlinear Drinfeld–Sokolov system is considered, in the following form, and is solved by the Exp function method

$$\begin{cases} u_t + (v^2)_x = 0, \\ v_t - av_{xxx} + 3bu_xv + 3cuv_x = 0. \end{cases} \quad (1)$$

where a , b , and c are constants. This system was introduced by Drinfeld and Sokolov as an example of a system of nonlinear equations possessing Lax pairs of a special form (Goktas and Hereman, 1997; Wazwaz, 2006).

^{*} Corresponding author.

E-mail address: biazar@guilan.ac.ir (J. Biazar).



Let us introduce a complex variable ξ , as follows

$$\xi = kx + wt. \quad (2)$$

So, Eq. (1) turns to the following system of ordinary different equation,

$$\begin{cases} wu' + k(v^2)' = 0, \\ wv' - akv''' + 3bku'v + 3ckuv' = 0. \end{cases} \quad (3)$$

where k and w are constant to be determined. A simplified form of the first equation will be derived by taking integral from both sides of that. Let us consider the integral constant zero.

$$u = -k \frac{v^2}{w}. \quad (4)$$

Substituting Eq. (4) into the second equation of the system and integrating lead to

$$w^2v - awk^3v'' - (2b + c)k^2v^3 = 0. \quad (5)$$

where c , d , p , and q are positive integers which could be freely chosen, a_m for $m = -d, \dots, c$ and b_n for $n = -q, \dots, p$ are unknown constants to be determined. To find the values of c and p , we balance the linear terms of the highest order in Eq. (8) with the highest order nonlinear terms.

Similarly to find out the values of d and q , we balance the linear terms of the lowest order in Eq. (8) with the lowest order nonlinear terms.

3. Exp function method for the DS system

The Exp function method as well addressed in He and Wu (2006), Zhang (2007), Biazar and Ayati (2008), and in this part it will be applied to obtain the solution of the Driinfeld–Sokolov system.

We assume that the solution of Eq. (5) can be expressed in the form shown in the following form

$$u(\xi) = \frac{a_c \exp(c\xi) + \dots + a_{-d} \exp(-d\xi)}{b_p \exp(p\xi) + \dots + b_{-q} \exp(-q\xi)}, \quad (6)$$

In order to determine the constants c and p , we balance the linear term of the highest order in Eq. (5) with the highest order nonlinear term. By simple calculation, we have

$$v'' = \frac{c_1 \exp[(3p + c)\xi] + \dots}{c_2 \exp[4p\xi] + \dots}, \quad (7)$$

and

$$v^3 = \frac{c_3 \exp[3c\xi] + \dots}{c_4 \exp[3p\xi] + \dots} = \frac{c_3 \exp[(p + 3c)\xi] + \dots}{c_4 \exp[4p\xi] + \dots}. \quad (8)$$

By balancing the highest order terms of Exp-functions in Eqs. (7) and (8), we have

$$c + 3p = 3c + p, \quad (9)$$

which leads to the result:

$$p = c. \quad (10)$$

Similarly, we balance the lowest order terms in Eq. (5) to determine values of d and q , we obtain:

$$d = q. \quad (11)$$

It is possible to choose the values of c and d , too.

3.1. The choice of $p = c = 1$, and $q = d = 1$

We choose $p = c = 1$, and $q = d = 1$, the trial function, Eq. (6) converts to the following form

$$v(\xi) = \frac{a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi)}{b_1 \exp(\xi) + b_0 + b_{-1} \exp(-\xi)}. \quad (12)$$

In the case $b_1 \neq 0$ Eq. (12) can be simplified as

$$v(\xi) = \frac{a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi)}{\exp(\xi) + b_0 + b_{-1} \exp(-\xi)}. \quad (13)$$

Substituting Eq. (13) into Eq. (5), and taking the coefficients of $\exp(n\xi)$ in each term zero yield to a set of algebraic equations for a_1 , a_0 , a_{-1} , b_0 , b_{-1} , k , and w . Solving this system of algebraic equations by the aid of Maple, or via any others, leads to

$$\begin{aligned} a_{-1} = 0, \quad a_0 = a_0, \quad a_1 = 0, \quad b_{-1} = \frac{1}{8} \frac{a_0^2(2b + c)}{k^4 a^2}, \quad b_0 \\ = 0, \quad k = k, \quad w = ak^3. \end{aligned} \quad (14)$$

where a_0 and k are free parameters. Substituting Eq. (14) into Eq. (13), we obtain the following exact solution

$$v(x, t) = \frac{a_0}{\exp(kx + ak^3 t) + \frac{1}{8} \frac{a_0^2(2b+c)}{k^4 a^2} \exp(-kx - ak^3 t)}. \quad (15)$$

If we set $a_0 = \frac{2\sqrt{2k^2 a}}{\sqrt{2b+c}}$, and $r = -ak^2$, Eq. (15) reduces to

$$v(x, t) = r \sqrt{\frac{2}{2b+c}} \sec h \left(\sqrt{\frac{-r}{a}} (x - rt) \right). \quad (16)$$

For $a_0 = \frac{2\sqrt{2k^2 a}}{\sqrt{2b+c}} i$, and $r = -ak^2$, we get

$$v(x, t) = -ir \sqrt{\frac{2}{2b+c}} \csc h \left(\sqrt{\frac{-r}{a}} (x - rt) \right). \quad (17)$$

These solutions are the same as the Wazwaz's solution Wazwaz's (2006) solution.

3.2. The choice of $p = c = 2$, and $q = d = 1$

If we choose $p = c = 2$, and $q = d = 1$, Eq. (6) takes the following form:

$$v(\xi) = \frac{a_2 \exp(2\xi) + a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi)}{\exp(2\xi) + b_1 \exp(\xi) + b_0 + b_{-1} \exp(-\xi)}. \quad (18)$$

Proceeding in a similar way as illustrated in Section 3.1, we can identify parameters, a_2 , a_1 , a_0 , a_{-1} , b_1 , b_0 , b_{-1} , w , and k in Eq. (39) as the following

$$\begin{aligned} a_2 = 0, \quad a_1 = a_1, \quad a_0 = 0, \quad a_{-1} = 0, \quad b_1 = 0, \\ b_0 = \frac{1}{8} \frac{a_1^2(2b+c)}{k^4 a^2}, \quad b_{-1} = 0, \quad w = ak^3, \quad k = k. \end{aligned} \quad (19)$$

$$\begin{aligned} a_2 = \pm \frac{1}{2} \frac{ak^2}{\sqrt{2b+c}}, \quad a_1 = \pm(1 + \sqrt{2}) \sqrt{\pm \frac{a_0 a}{\sqrt{2b+c}}} k, \quad a_0 = a_0, \\ a_{-1} = 0, \quad b_1 = 0, \quad b_0 = \pm \frac{2a_0}{k^2 a} \sqrt{2b+c}, \quad (20) \\ b_{-1} = 0, \quad w = -\frac{1}{2} ak^3, \quad k = k. \end{aligned}$$

Substituting Eqs. (19) and (20) into Eq. (18), we obtain the following exact solutions;

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